ECLECTIC EDUCATION SERIES

Ray's Surveying and Navigation

RAY'S MATHEMATICAL SERIES.

SURVEYING

AND

NAVIGATION,

WITH A PRELIMINARY TREATISE ON

TRIGONOMETRY AND MENSURATION,

BY

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***Higher Arithmetic**, **Principles** of **Logic**, and **Complete Algebra**.*

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PREFACE.

Nearly twenty years ago the Publishers made the following announcement: "*Surveying and Navigation;* containing Surveying and Leveling, Navigation, Barometric Heights, etc."

To redeem this promise, the present work now appears.

It is customary to preface works on Surveying by a meager sketch of Plane Trigonometry, but it has been thought best to include in this work a thorough treatment of Plane and Spherical Trigonometry and Mensuration. These subjects have been treated in view of the wants of our best High Schools and Colleges.

Certain modern writers have defined the Trigonometric functions as ratios; for example, in a right triangle, the sine of an angle is the ratio of the opposite side to the hypotenuse, etc.

The historical method of considering the sine, co-sine, tangent, etc., as linear functions of the arc, explains the origin of these terms — avoids the ambiguity of the word *ratio;* explains how the logarithm of the sine, for example, can reach the limit 10, which would be impossible if the limit of the sine itself is 1, and is much more readily apprehended by the student.

The advantages in analytic investigations resulting from defining these functions as ratios have been secured in the principles relating to the Right Triangle, Art. 64.

Each of the circular functions has, in the first place, been considered by itself, and its value traced, for all arcs, from 0° to 360°.

Then follows the solution of triangles, right and oblique, the general relations of the circular functions, the functions of the sum or difference of two angles, and a variety of interesting practical applications.

It is hoped that Spherical Trigonometry has been made intelligible to the diligent student. More than ordinary care has been given to the development of Napier's principles, and to the discussion of the species of the parts of both right and oblique spherical triangles, Arts. 126, 129, 145, 148, 151.

Mensuration, a subject at once interesting and practically important, has been discussed at length, and formulas have been developed instead of rules for the solution of the problems.

In the Surveying, the instruments are first represented and described, and the methods of making the adjustments given in detail.

The Author takes this opportunity to express his obligations to Messrs. W. & L. E. Gurley, Manufacturers of Surveying and Engineering Instruments, Troy, N. Y., who have kindly granted him the use of their Manual for the delineation and description of the instruments. In consequence of this courtesy, much better drawings and descriptions have been made than would otherwise have been possible.

The instruments themselves should, however, be accessible to the student, who should study them in connection with the descriptions in the book, and learn to use them in practical work, guided by a competent instructor.

The Rectangular method of surveying the Public lands, now brought to great perfection under the direction of the Government, has been minutely explained, and illustrated by field notes of actual surveys. In this portion of the work, the United States Manual of Surveying Instructions has been taken as authority, and thus the authorized methods, which must form the basis for subsequent surveys, have been made accessible to the student.

The methods of finding the true meridian and the variation of the needle have been given at length; also specific directions for finding corners, taking bearings, measuring lines, recording field notes, and plotting.

In addition to the ordinary method of finding the area, a new method, developed by E. M. Pogue, of Kentucky, is given in Art. 304. This method has the merit of giving always a uniform result from the same field notes, and thus avoids disputes about the different results of the ordinary method, unavoidably attending the various distribution of errors by different calculators.

The methods of supplying omissions are explained and illustrated by examples.

Laying out and dividing land, operations admitting of an unlimited variety of applications, have been treated in view of the wants of the practical surveyor. The subject is also full of interest to the student, who can not fail to receive from it new views of the resources of mathematical science.

Leveling, the construction of railroad curves, embankments and excavations, the method of making Topographical surveys, with the authorized conventional symbols, Barometric heights, etc., have been explained and illustrated by diagrams and examples.

It has been thought best to give a clear, elementary treatment of Navigation, not only on account of those who may desire to pursue the subject further, but for the sake of gratifying the wishes of intelligent persons who may desire to know something of Navigation. The limits of the work, however, forbid the discussion of Nautical Astronomy. The examples in Navigation have been selected from the English work of J. R. Young.

The tables of Logarithms, Natural and Logarithmic sines, etc., have been carried only to five decimal places, and for the purposes intended will be found practically better than tables to six or seven places.

The Traverse table has been thrown into a new form, at once condensed and convenient.

These tables have been compiled by Mr. Henry H. Vail, and by him compared with Babbage's and Wittstein's tables, then by the Author with Vega's tables to seven decimal places. It is hoped that by this double comparison perfect accuracy has been attained.

The table of Meridional Parts, taken from "Projection Tables for the use of the United States Navy," prepared by the Bureau of Navigation, and issued from the Government Printing office, was calculated in the Hydrographic office for the terrestrial spheroid, compression $\frac{1}{299.1528}$. This table, now for the first time published in a text-book, is believed to be more correct than those in general use.

The Author takes pleasure in acknowledging his obligations to Prof. E. H. Warner for critical suggestions and acceptable aid in reading proof and testing the accuracy of the answers.

With the hope that the book will be attractive and useful to the student, teacher, and practical surveyor, it is sent forth to accomplish its work.

A. SCHUYLER.

BALDWIN UNIVERSITY,
BEREA, O., June 12, 1873.

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INTRODUCTION.

LOGARITHMS.

1. Definition.

A logarithm of a number is the exponent denoting the power to which a fixed number, called the base, must be raised in order to produce the given number.

Thus, in the equation, $b^x = n$, b is the base of the system, n is the number whose logarithm is to be taken, and x is the logarithm of n to the base b, which may be written: $x = \log_b n$.

Any positive number, except 1, may be assumed as the base, but when assumed, it remains fixed for a system; hence, there may be an infinite number of systems, since there may be an infinite number of bases.

2. Common Logarithms.

Common logarithms are the logarithms of numbers in the system whose base is 10.

$$
\begin{array}{lll}
10^0 = 1; & \therefore & \text{by def., } \log 1 = 0. \\
10^1 = 10; & \therefore & \text{by def., } \log 10 = 1. \\
10^2 = 100; & \therefore & \text{by def., } \log 100 = 2. \\
10^3 = 1000; & \therefore & \text{by def., } \log 1000 = 3. \\
\ldots & & \ldots
\end{array}
$$

Hence, *In the common system, the logarithm of an exact power of* 10 *is the whole number equal to the exponent of the power.*

3. Consequences.

1. If the number is greater than 1 and less than 10, its logarithm is greater than 0 and less than 1, or is 0 + a decimal.

2. If the number is greater than 10 and less than 100, its logarithm is greater than 1 and less than 2, or is 1 + a decimal.

3. In general, if the number is not an exact power of 10, its logarithm, in the common system, will consist of two parts—an entire part and a decimal part.

The entire part is called the *characteristic* and the decimal part is called the *mantissa.*

4. Problem.

To find the laws for the characteristic.

Let (1) $10^x = n$; then, by def., $\log n = x$.
But (2) $10^1 = 10$.

$$(1) \div (2) = (3)\ 10^{x-1} = \frac{n}{10};\ \text{then, by def., } \log \frac{n}{10} = x - 1.$$

$$\therefore \quad \log \frac{n}{10} = \log n - 1.$$

Hence, *The logarithm of the quotient of any number by* 10 *is less by* 1 *than the logarithm of the number.*

Let us now take the number 8979 and its logarithm 3.95323, as given in a table of logarithms, and divide the number successively by 10, and for each division subtract 1 from the logarithm of the dividend; then we have,

Log	8979 $= 3.95323$.	Log	.8979 $= \bar{1}.95323$.
"	897.9 $= 2.95323$.	"	.08979 $= \bar{2}.95323$.
"	89.79 $= 1.95323$.	"	.008979 $= \bar{3}.95323$.
"	8.979 $= 0.95323$.	. . . .	

The minus sign applies only to the characteristic over which it is placed.

The mantissa is always positive, and is the same for all positions of the decimal point.

An inspection of the above will reveal the following laws:

1. *If the number is integral or mixed, the characteristic is positive and is one less than the number of integral figures.*

2. *If the number is entirely decimal, the characteristic is negative and is one greater, numerically, than the number of 0's immediately following the decimal point.*

5. Exercises on the Characteristic.

1. What is the characteristic of the logarithm of 7?

2. What is the characteristic of the logarithm of 465?

3. What is the characteristic of the logarithm of 4678?

4. What is the characteristic of the logarithm of 34.75?

5. What is the characteristic of the logarithm of .65?

6. What is the characteristic of the logarithm of .0789?

7. What is the characteristic of the logarithm of .00084?

8. If the characteristic of the logarithm of a number is 2, how many integral places has that number?

9. If the characteristic of the logarithm of a number is 5, how many integral places has that number?

10. If the characteristic of the logarithm of a number is 1, how many integral places has that number?

11. If the characteristic of the logarithm of a number is 0, how many integral places has that number?

12. If the characteristic of the logarithm of a number is negative, is the number integral, decimal, or mixed?

13. If the characteristic of the logarithm of a number is $\overline{4}$, how many 0's immediately follow the decimal point?

14. If the characteristic of the logarithm of a number is $\overline{2}$, how many 0's immediately follow the decimal point?

15. If the characteristic of the logarithm of a number is $\overline{1}$, how many 0's immediately follow the decimal point?

TABLE OF LOGARITHMS.

6. Description of the Table.

The table of logarithms annexed gives the mantissa of the logarithm of every number from 1000 to 10900. The characteristic can be found by the preceding laws.

It follows, from Art. 4, that the mantissa of the logarithm of a number is the same as the mantissa of the logarithm of the product or quotient of that number by any power of 10. Thus:

$$\begin{aligned} \text{Log } 12 &= 1.07918. \\ \text{“ } 120 &= 2.07918. \\ \text{“ } .012 &= \overline{2}.07918. \end{aligned}$$

Hence, we can determine from the table the logarithm of any number less than 1000. Thus, the mantissa of the logarithm of 8 is the same as that of the logarithm of 8000.

In the table, the first three or four figures of each number are given in the left-hand column, marked *N*. The next figure is given at the head and foot of one of the columns of mantissas.

The mantissas, in the column under 0, are given to five decimal places. The first and second decimal figures of this column are understood to be repeated in the spaces below, and to be prefixed, across the page, to the three figures of the remaining columns.

When the third decimal digit changes from 9 to 0, the second is increased by the 1 carried; and the corresponding mantissa, and all to the right, commence with a smaller figure, to indicate that the first two decimal figures, to be prefixed, are to be taken from the line below.

The last column, marked D, contains the difference of two successive mantissas, called the *tabular difference.*

7. Problem.

To find the logarithm of a given number.

1. Find the logarithm of 3675.

The characteristic is 3. Opposite 367, in the column headed N, and under the column headed 5, we find 526, to which prefix the two figures, 56, in the column headed 0, and we have for the mantissa .56526.

$$\therefore \log 3675 = 3.56526.$$

2. Find the logarithm of 76.

The characteristic is 1, and the mantissa is the same as that of 7600, which is .88081.

$$\therefore \log 76 = 1.88081.$$

3. Find the logarithm of .004268.

The characteristic is $\overline{3}$, and the mantissa is the same as that of 4268. Looking opposite 426, and under 8, we find 022, of which the 0 is a small figure. Prefixing

63, from the line below, in the column headed 0, we have for the mantissa .63022.

$$\therefore \quad \log .004268 = \bar{3}.63022.$$

4. Find the logarithm of 109684.

$$\begin{array}{lr} \text{The characteristic} & = 5. \\ \text{The mantissa of log } 1096 \quad = & .03981 \\ \text{Tab. diff. is 40; and } 40 \times .84 = & 34 \\ \hline \log \ 109684 = & 5.04015 \end{array}$$

The reason for multiplying the tabular difference by .84 will be apparent from the following:

$$\log \ 109600 = 5.03981.$$
$$\log \ 109700 = 5.04021.$$

The difference of the logarithms is 40 hundred-thousandths, and the difference of the numbers is 100; but the difference of 109600 and 109684 is 84, which is .84 of 100; hence, the difference of the logarithms of 109600 and 109684 is .84 of 40 hundred-thousandths, which is 40 hundred-thousandths $\times$ .84 = 34 hundred-thousandths, nearly.

It is assumed that the difference of the logarithms of two numbers is proportional to the difference of the numbers, which is approximately true, especially if the numbers are large.

5. Find the logarithm of 123.613.

$$\begin{array}{lr} \text{The characteristic} & = 2. \\ \text{The mantissa of log } 1236 \quad = & .09202 \\ \text{Tab. diff. is 35; and } 35 \times .13 = & 5 \\ \hline \therefore \quad \log \ 123.613 = & 2.09207 \end{array}$$

The tabular difference is .00035, and .00035 $\times$ .13 = .0000455. But since the logarithms in this table are taken only to five decimal places, the two last figures, 55, are rejected, and 1 is carried to .00004, making .00005 for the correction.

In general, when the left-hand figure of the part rejected exceeds 4, carry 1.

When the tabular difference is large, as in the first part of the table, there may be small errors. Accordingly, for numbers between 10000 and 10900, it will be better to use the last two pages instead of the first page.

8. Rule.

1. *If the number, or the product of the number by any power of* 10, *is found in the table, take the corresponding mantissa from the table, and prefix the proper characteristic.*

2. *If the number, without reference to the decimal point or* 0's *on the right, is expressed by more than five figures, take from the table the mantissa corresponding to the first four or five figures on the left, multiply the corresponding tabular difference by the number expressed by the remaining figures, considered as a decimal, reject from the product as many figures on the right as are in the multiplier, carrying to the nearest unit, and add the result as so many hundred-thousandths to the mantissa before found, and to the sum prefix the proper characteristic.*

9. Examples.

1. What is the logarithm of 2347? *Ans.* 3.37051.
2. What is the logarithm of 108457? *Ans.* 5.03526.
3. What is the logarithm of 376542? *Ans.* 5.57581.
4. What is the logarithm of 229.7052? *Ans.* 2.36117.
5. What is the logarithm of 1128737? *Ans.* 6.05260.
6. What is the logarithm of .30365? *Ans.* $\bar{1}.48237$.
7. What is the logarithm of .0042683? *Ans.* $\bar{3}.63025$.
8. What is the logarithm of 1245400? *Ans.* 6.09531.

10. Problem.

To find the number corresponding to a given logarithm.

1. What number corresponds to logarithm $\bar{2}.03262$?

The mantissa is found in the column headed 8, and opposite 107 in the column headed N. Hence, without reference to the decimal point, the number corresponding is 1078; but since the characteristic is $\bar{2}$, the number is entirely decimal, and one 0 immediately follows the decimal point. Hence, the number corresponding is .01078.

2. What number corresponds to logarithm 2.83037?

Since this logarithm can not be found in the table, take the next less, which is 2.83033, and the corresponding number, without reference to the decimal point, which is 6766.

The difference between the given logarithm and the next less is 4, and the tabular difference is 6, which is the difference of the logarithms of the two numbers, 6766 and 6767, whose difference is 1.

If the tabular difference of the logarithms, 6, corresponds to a difference in the numbers of 1, the difference of the logarithms, 4, will correspond to a difference of $\frac{4}{6}$ of 1; which, reduced to a decimal, and annexed to 6766, will give for the number, without reference to the decimal point, 676666. But since the characteristic is 2, there will be three integral places; hence, 676.666 is the number required.

3. What number corresponds to logarithm 2.76398?

$$
\begin{array}{lll}
\text{The given log} = 2.76398 & \therefore \text{ number} = 580.737 \\
\text{Next less log} = \underline{2.76395} & \therefore \text{ number} = 580.7 \\
\qquad \text{Tab. difference} = 8)\underline{300} = \text{difference.} & \\
\qquad \qquad \qquad \qquad 37 = \text{correction.} &
\end{array}
$$

It is necessary to write only that part of the next less logarithm which differs from the given logarithm. Conceive 0's annexed to the difference, and divide by the tabular difference; and annex the quotient to the number corresponding to the next less logarithm.

In practical work abbreviate thus: Let l denote the given logarithm; l', the next less logarithm; n and n', the corresponding numbers; t, the tabular difference; d, difference of logarithms; c, the correction.

4. What number corresponds to logarithm $\bar{1}.73048$?

$$\begin{array}{ll} l = \bar{1}.73048 & \therefore\ n = .537625 \\ l' = \bar{1}.73046 & \therefore\ n' = .5376 \\ t = 8)\underline{2} = d. & \\ \quad 25 = c. & \end{array}$$

n' is found first, then n by annexing c.

11. Rule.

1. *If the given mantissa can be found in the table, take the number corresponding, and place the decimal point according to the law for the characteristic.*

2. *If the given mantissa can not be found in the table, take the next less and the corresponding number. Subtract this mantissa from the given mantissa, annex 0's to the remainder, divide the result by the tabular difference, annex the quotient to the number corresponding to the logarithm next less than the given logarithm, and place the decimal point according to the law for the characteristic.*

12. Examples.

1. What number corresponds to logarithm 4.55703?

Ans. 36060.

2. What number corresponds to logarithm 3.95147?

Ans. 8942.8.

3. What number corresponds to logarithm $\bar{2}.41130$?

Ans. .025781.

4. What number corresponds to logarithm $\bar{1}.48237$?

Ans. .30365.

5. What number corresponds to logarithm $\bar{3}.63025$?

Ans. .0042683.

MULTIPLICATION BY LOGARITHMS.

13. Proposition.

The logarithm of the product of two numbers is equal to the sum of their logarithms.

$$\text{Let } \begin{cases} (1)\ b^x = m;\ \text{then, by def., } \log m = x. \\ (2)\ b^y = n;\ \text{then, by def., } \log n = y. \end{cases}$$

$$(1)\times(2) = (3)\ b^{x+y} = m\,n;\ \text{then, by def., } \log m\,n = x+y.$$

$$\therefore\ \log m\,n = \log m + \log n.$$

14. Rule.

1. *Find the logarithms of the factors and take their sum, which will be the logarithm of the product.*

2. *Find the number corresponding which will be their product.*

15. Examples.

1. Find the product of 57846 and .003927.

$$\begin{aligned} \log 57846 &= 4.76228 \\ \log .003927 &= \underline{\bar{3}.59406} \\ \log \text{product} &= 2.35634, \qquad \therefore\ \text{product} = 227.16. \end{aligned}$$

2. Find the product of 37.58 and 75864.

Ans. 2851000.

3. Find the product of .3754 and .00756.

Ans. .002838.

4. Find the product of 999.75 and 75.85.

Ans. 75831.667.

5. Find the product of 85, .097, and .125. *Ans.* 1.03062.

DIVISION BY LOGARITHMS.

16. Proposition.

The logarithm of the quotient of two numbers is equal to the logarithm of the dividend minus the logarithm of the divisor.

$$\text{Let}\begin{cases}(1) & b^x = m;\ \text{then, by def., } \log m = x.\\ (2) & b^y = n;\ \text{then, by def., } \log n = y.\end{cases}$$

$$(1) \div (2) = (3) \quad b^{x-y} = \frac{m}{n};\quad \text{then, by def., } \log \frac{m}{n} = x - y.$$

$$\therefore \quad \log \frac{m}{n} = \log m - \log n.$$

17. Rule.

1. *Find the logarithms of the numbers, subtract the logarithm of the divisor from the logarithm of the dividend, and the remainder will be the logarithm of the quotient.*

2. *Find the number corresponding which will be the quotient.*

18. Examples.

1. Divide 73.125 by .125.

$$\begin{aligned}\log \quad 73.125 &= 1.86407\\ \log \quad .125 &= \bar{1}.09691\\ \hline \log \text{ quotient} &= 2.76716, \qquad \therefore \text{ quotient} = 585.\end{aligned}$$

2. Divide 7.5 by .000025. *Ans.* 300000.
3. Divide 87.9 by .0345. *Ans.* 2547.824.
4. Divide .34852 by .00789. *Ans.* 44.171.
5. Divide 85734 by 12.7523. *Ans.* 6723.