ECLECTIC EDUCATION SERIES

Ray's New Elementary Algebra

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NEW ELEMENTARY ALGEBRA.

PRIMARY ELEMENTS

OF

ALGEBRA,

FOR

COMMON SCHOOLS AND ACADEMIES.

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REVISED ELECTROTYPE EDITION.

VAN ANTWERP, BRAGG & CO.,

CINCINNATI: 137 WALNUT ST.

NEW YORK: 28 BOND ST.

Ray's Mathematical Series.

Embracing a Thorough, Progressive, and Complete Course

in Arithmetic, Algebra, and the Higher Mathematics.

PREFACE.

THE object of the study of Mathematics is two fold—the acquisition of useful knowledge, and the cultivation and discipline of the mental powers. A parent often inquires, "Why should my son study Mathematics? I do not expect him to be a surveyor, an engineer, or an astronomer." Yet, the parent is very desirous that his son should be able to reason correctly, and to exercise, in all his relations in life, the energies of a cultivated and disciplined mind. This is, indeed, of more value than the mere attainment of any branch of knowledge.

The science of Algebra, properly taught, stands among the first of those studies essential to both the great objects of education. In a course of instruction properly arranged, it naturally follows Arithmetic, and should be taught immediately after it.

In the following work, the object has been to furnish an elementary treatise, commencing with the first principles, and leading the pupil, by gradual and easy steps, to a knowledge of the elements of the science. The design has been, to present these in a brief, clear, and scientific manner, so that the pupil should not be taught merely to perform a certain routine of exercises mechanically, but to understand the *why* and the *wherefore* of every step. For this purpose, every rule is demonstrated, and every principle analyzed, in order that the mind of the pupil may be disciplined and strengthened so as to prepare him, either for pursuing the study of Mathematics intelligently, or more successfully attending to any pursuit in life.

Some teachers may object, that this work is too simple, and too easily understood. A leading object has been, to make the pupil feel, that he is not operating on unmeaning symbols, by means of arbitrary rules; that Algebra is both a rational and a practical subject, and that he can rely upon his reasoning, and the results of his operations, with the same confidence as in arithmetic. For this purpose, he is furnished, at almost every step, with the means of testing the accuracy of the principles on which the rules are founded, and of the results which they produce.

Throughout the work, the aim has been to combine the clear explanatory methods of the French mathematicians with the practical exercises of the English and German, so that the pupil should acquire both a practical and theoretical knowledge of the subject.

While every page is the result of the author's own reflection, and the experience of many years in the school-room, it is also proper to state, that a large number of the best treatises on the same subject, both English and French, have been carefully consulted, so that the present work might embrace the modern and most approved methods of treating the various subjects presented.

With these remarks, the work is submitted to the judgment of fellow laborers in the field of education.

WOODWARD COLLEGE, *August*, 1848.

In this NEW ELECTROTYPE EDITION, the whole volume has been subjected to a careful and thorough revision. The oral problems, at the beginning, have been omitted; the number of examples reduced, where they were thought to be needlessly multiplied; the rules and demonstrations abridged; other methods of proof, in a few instances, substituted; and questions for GENERAL REVIEW introduced at intervals, and at the conclusion. It is confidently believed that these modifications, while they do not impair the integrity or change the essential features of the book, will materially enhance its value, and secure the approbation of all intelligent teachers.

March, 1866.

TO TEACHERS.—The following subjects may be omitted by the younger pupils, and passed over by those more advanced, until the book is reviewed: Observations on Addition and Subtraction, Articles 60—64; the greater part of Chapter II.; supplement to Simple Equations, Articles 164—177; properties of the Roots of an Equation of the Second Degree, Articles 215—217.

The pupil should be exercised in the solution of examples, until the principles are thoroughly understood; and, in the review, he should be required to demonstrate the rules on the blackboard.

CONTENTS.

I.—FUNDAMENTAL RULES.

II.—THEOREMS, FACTORING, ETC.

III.—ALGEBRAIC FRACTIONS.

IV.—SIMPLE EQUATIONS.

ELEMENTS OF ALGEBRA.

I. DEFINITIONS.

Note to Teachers.—Articles 1 to 15 may be omitted until the pupil reviews the book.

Article 1. In **Algebra,** quantities are represented by letters of the alphabet.

2. Quantity is any thing that is capable of increase or decrease; as, numbers, lines, space, time, etc.

3. Quantity is called *magnitude,* when considered in an undivided form; as, a quantity of water.

4. Quantity is called *multitude,* when made up of individual and distinct parts; as, three cents, a quantity composed of three single cents.

5. One of the single parts of which a quantity of multitude is composed, is called the *unit of measure;* thus, 1 cent is the *unit of measure* of the quantity 3 cents.

The *value* or *measure* of any quantity is the number of times it contains its unit of measure.

6. In quantities of magnitude, where there is no natural unit, it is necessary to fix upon an artificial unit as a standard of measure; then, to find the value of the quantity, we ascertain how many times it contains its *unit of measure.* Thus,

To measure the length of a line, take a certain assumed

Review.—1. How are quantities represented in Algebra? 2. What is quantity? 3. When called magnitude?
4. When multitude? 5. What is the unit of measure? 6. How find the value of a quantity when there is no natural unit?

distance called a foot, and, applying it a certain number of times, say 5, it is found that the line is 5 feet long; in this case, 1 foot is the *unit of measure.*

7. The **Numerical Value** of a quantity is the number that shows how many times it contains its unit of measure.

Thus, the numerical value of a line 5 feet long, is 5. The same quantity may have different numerical values, according to the unit of measure assumed.

8. A **Unit** is a single thing of an order or kind.

9. **Number** is an expression denoting a unit, or a collection of units. Numbers are either abstract or concrete.

10. An **Abstract Number** denotes how many times a unit *is to be taken.*

A **Concrete Number** denotes the units that *are taken.*

Thus, 4 is an abstract number, denoting merely the number of units taken; while 4 feet is a concrete number, denoting what unit is taken, as well as the number taken.

Or, *a concrete number is the product of the unit of measure by the corresponding abstract number.* Thus, $6 equal $1 multiplied by 6, or $1 taken 6 times.

11. In algebraic computations, *letters* are considered the representatives of *numbers.*

12. There are two kinds of questions in Algebra, *theorems* and *problems.*

13. In a **Theorem,** it is required to demonstrate some relation or property of numbers, or abstract quantities.

14. In a **Problem,** it is required to find the value of some *unknown* quantity, by means of certain given relations existing between it and others, which are *known.*

REVIEW.—7. Define numerical value. 8. What is a unit? 9. A number? 10. What does an abstract number denote? A concrete number? 11. What do the letters used in Algebra represent? 12. How many kinds of questions in Algebra? 13. What is a theorem? 14. A problem?

15. **Algebra** is a general method of solving problems and demonstrating theorems, by means of *figures*, *letters*, and *signs*. The letters and signs are called *symbols*.

EXPLANATION OF SIGNS AND TERMS.

16. **Known Quantities** are those whose values are given; **Unknown Quantities,** those whose values are to be determined.

17. Known quantities are generally represented by the first letters of the alphabet, as a, b, c, etc.; unknown quantities, by the last letters, as x, y, z.

18. The principal signs used in Algebra are

$$=,\quad +,\quad -,\quad \times,\quad \div,\quad (\ ),\quad >,\quad \sqrt{\ }.$$

Each sign is the representative of certain words. They are used to express the various operations in the clearest and briefest manner.

19. The **Sign of Equality,** $=$, is read *equal to.* It denotes that the quantities between which it is placed are equal. Thus, $a=3$, denotes that the quantity represented by a is equal to 3.

20. The **Sign of Addition,** $+$, is read *plus.* It denotes that the quantity to which it is prefixed is to be added.

Thus, $a+b$, denotes that b is to be added to a. If $a=2$ and $b=3$, then $a+b=2+3$, which $=5$.

21. The **Sign of Subtraction,** $-$, is read *minus.* It denotes that the quantity to which it is prefixed is to be subtracted.

Thus, $a-b$, denotes that b is to be subtracted from a. If $a=5$ and $b=3$, then $5-3=2$.

REVIEW.—15. What is Algebra? What are symbols? 16. What are known quantities? Unknown quantities? 17. By what are known quantities represented? Unknown quantities?

18. Write the principal signs used in Algebra. What does each represent? For what used?

19. How is the sign $=$ read? What does it denote? 20. How is the sign $+$ read? What denote? 21. How is the sign $-$ read? What denote?

22. The signs $+$ and $-$ are called *the* signs. The former is called the *positive*, the latter the *negative* sign: they are said to be *contrary* or *opposite.*

23. Every quantity is supposed to be preceded by one of these signs. Quantities having the positive sign are called *positive;* those having the negative sign, *negative.*

When a quantity has no sign prefixed, it is positive.

24. Quantities having the same sign are said to have *like* signs; those having different signs, *unlike* signs.

Thus, $+a$ and $+b$, or $-a$ and $-b$, have like signs; while $+c$ and $-d$ have unlike signs.

25. The **Sign of Multiplication,** $\times$, is read *into*, or *multiplied by.* It denotes that the quantities between which it is placed are to be multiplied together.

The product of two or more letters is sometimes expressed by a dot or point, but more frequently by writing them in close succession without any sign. Thus, ab expresses the same as $a\times b$ or $a.b$, and $abc=a\times b\times c$, or $a.b.c.$

26. Factors are quantities that are to be multiplied together.

The *continued product* of several factors means the product of the first and second multiplied by the third, this product by the fourth, and so on.

Thus, the continued product of a, b, and c, is $a\times b\times c$, or abc. If $a=2$, $b=3$, and $c=5$, then $abc=2\times 3\times 5=6\times 5=30$.

27. The **Sign of Division,** $\div$, is read *divided by.* It

REVIEW.—22. What are the signs plus and minus called, by way of distinction? Which is positive? Which negative?

23. What are quantities preceded by the sign plus said to be? By the sign minus? When no sign is prefixed? 24. When do quantities have like signs? When unlike signs?

25. How is the sign $\times$ read, and what does it denote? What other methods of representing multiplication? 26. What are factors? How many in a? In ab? In abc? In $5abc$?

27. How is the sign $\div$ read, and what does it denote? What other methods of representing division?

denotes that the quantity preceding it is to be divided by that following it. Division is oftener represented by placing the dividend as the numerator, and the divisor as the denominator of a fraction.

Thus, $a \div b$, or $\frac{a}{b}$, means, that a is to be divided by b. If $a=12$ and $b=3$, then $a \div b=12 \div 3=4$; or $\frac{a}{b}=\frac{12}{3}=4$.

Division is also represented thus, $a\rfloor b$, or $b\lfloor a$, a denoting the dividend, and b the divisor.

28. The **Sign of Inequality,** $>$, denotes that one of the two quantities between which it is placed is greater than the other. The *opening* of the sign is toward the *greater* quantity.

Thus, $a>b$, denotes that a is greater than b. It is read, a greater than b. If $a=5$, and $b=3$, then $5>3$. Also, $c<d$, denotes that c is less than d. It is read, c less than d. If $c=4$ and $d=7$, then $4<7$.

29. The **Sign of Infinity,** ∞, denotes a quantity greater than any that can be assigned, or one indefinitely great.

30. The **Numeral Coefficient** of a quantity is a number prefixed to it, showing how many times the quantity is taken.

Thus, $a+a+a+a=4a$; and $ax+ax+ax=3ax$.

31. The **Literal Coefficient** of a quantity is a quantity by which it is multiplied. Thus, in the quantity ay, a may be considered the coëfficient of y, or y the coëfficient of a.

The literal coëfficient is generally regarded as a known quantity.

32. The coëfficient of a quantity may consist of a number and a literal part. Thus, in $5ax$, $5a$ may be re-

REVIEW.—28. What is the sign $>$ called, and what does it denote? Which quantity is placed at the opening?
29. What does the sign ∞ denote? 30. What is a numeral coëfficient? How often is ax taken in $3ax$? In $5ax$? In $7ax$?
31. What is a literal coëfficient? 32. When a quantity has no coëfficient, what is understood?

garded as the coëfficient of x. If $a=2$, then $5a=10$, and $5ax=10x$.

When no numeral coëfficient is prefixed to a quantity, its coëfficient is understood to be unity. Thus, $a=1a$, and $bx=1bx$.

33. The **Power** of a quantity is the product arising from multiplying the quantity by itself one or more times.

When the quantity is taken twice as a factor, the product is called its *square*, or *second* power; when three times, the *cube*, or *third* power; when four times, the *fourth* power, and so on.

Thus, $a\times a=aa$, is the *second* power of a; $a\times a\times a=aaa$, is the *third* power of a; $a\times a\times a\times a=aaaa$, is the *fourth* power of a.

An **Exponent** is a figure placed at the right, and a little above a quantity, to show how many times it is taken as a factor.

Thus, $aa=a^2$; $aaa=a^3$; $aaaa=a^4$; $aabbb=a^2b^3$.

When no exponent is expressed, it is understood to be unity. Thus, a is the same as a^1, each expressing the first power of a.

34. To raise a quantity to any given power is to find that power of the quantity.

35. The **Root** of a quantity is another quantity, some power of which equals the given quantity. The root is called the square root, cube root, fourth root, etc., according to the number of times it is taken as a factor to produce the given quantity.

Thus, a is the second or square root of a^2, since $a\times a=a^2$. So, x is the third or cube root of x^3, since $x\times x\times x=x^3$.

36. To *extract* any root of a quantity is to find that root.

REVIEW.—33. What is the power of a quantity? What is the second power of a? The third power of a?

33. What is an exponent? For what used? How many times is x taken as a factor in x^2? In x^3? In x^5? Where no exponent is written, what is understood? 35. What is the root of a quantity?

37. The **Radical Sign,** $\sqrt{}$, placed before a quantity, indicates that its root is to be extracted.

Thus, $\sqrt[2]{a}$, or $\sqrt{a}$, denotes the square root of a; $\sqrt[3]{a}$, denotes the cube root of a; $\sqrt[4]{a}$, denotes the fourth root of a.

38. The number placed over the radical sign is called the *index* of the root. Thus, 2 is the index of the square root, 3 of the cube root, 4 of the fourth root, and so on. When the radical has no index over it, 2 is understood.

39. Every quantity or combination of quantities expressed by means of symbols, is called an *algebraic expression.*

Thus, $3a$ is the algebraic expression for 3 times the quantity a; $3a-4b$, for 3 times a, diminished by 4 times b; $2a^2+3ab$, for twice the square of a, increased by 3 times the product of a and b.

40. A **Monomial,** or **Term,** is an algebraic expression, not united to any other by the sign $+$ or $-$.

A monomial is sometimes called a *simple quantity.* Thus, a, $3a$, $-a^2b$, $2any^2$, are monomials, or simple quantities.

41. A **Polynomial** is an algebraic expression, composed of two or more terms.

Thus, $c+2d-b$ is a polynomial.

42. A **Binomial** is a polynomial composed of two terms. Thus, $a+b$, $a-b$, and c^2-d, are binomials.

A **Residual Quantity** is a binomial, in which the second term is negative, as $a-b$.

43. A **Trinomial** is a polynomial consisting of *three terms.* Thus, $a+b+c$, and $a-b-c$, are trinomials.

44. The **Numerical Value** of an algebraic expression

REVIEW.—37. What is the sign $\sqrt{}$ called, and what does it denote? 38. What is the number placed over the radical sign called? 39. What is an algebraic quantity? 40. A monomial? A simple quantity? 41. A polynomial? 42. A binomial? A residual quantity?
43. A trinomial? 44. What is the numerical value of an algebraic expression?

is the number obtained, by giving particular values to the letters, and then performing the operations indicated.

In the algebraic expression $2a+3b$, if $a=4$, and $b=5$, then $2a=8$, and $3b=15$, and the numerical value is $8+15=23$.

45. The value of a polynomial is not affected by changing the order of the terms, provided each term retains its respective sign. Thus, $a^2+2a+b=b+a^2+2a$. This is self-evident.

46. Each of the *literal* factors of any simple quantity or term is called a *dimension* of that term. The *degree* of a term depends on the number of its literal factors.

Thus, ax consists of two literal factors, a and x, and is of the *second degree.* The quantity a^2b contains three literal factors, a, a, and b, and is of the *third degree.* $2a^3x^2$ contains 5 literal factors, a, a, a, x, and x, and is of the *fifth degree;* and so on.

47. A polynomial is said to be *homogeneous,* when each of its terms is of the same degree.

Thus, the polynomials $2a-3b+c$, of the first degree, $a^2+3bc+xy$, of the second degree, and x^3-8ay^2, of the third degree, are homogeneous: a^3+x^2 is not homogeneous.

48. A **Parenthesis,** ( ), is used to show that all the included terms are to be considered together as a single term.

Thus, $4\,(a-b)$ means that $a-b$ is to be multiplied by 4; $(a+x)\,(a-x)$ means that $a+x$ is to be multiplied by $a-x$; $10-(a+c)$ means that $a+c$ is to be subtracted from 10; $(a-b)^2$ means that $a-b$ is to be raised to the second power; and so on.

49. A **Vinculum,** ———, is sometimes used instead of

REVIEW.—46. What is the dimension of a term? On what does the degree of a term depend? What is the degree of the term xy? Of xyz? Of $2axy$? Of x^2? 47. When is a polynomial homogeneous? 48. For what is a parenthesis used? 49. What is a vinculum, and for what used?

a parenthesis. Thus, $\overline{a-b}\times x$ means the same as $(a-b)x$. Sometimes the vinculum is placed vertically: it is then called a *bar*.

Thus, $\left.\begin{array}{r} a \\ -x \\ +4 \end{array}\right| y^2$ has the same meaning as $(a-x+4)y^2$.

50. **Similar** or **Like** quantities are those composed of the same letters, affected with the same exponents.

Thus, $7ab$ and $-3ab$, also $4a^3b^2$ and $7a^3b^2$, are similar terms; but $2a^2b$ and $2ab^2$ are not similar; for, though composed of the same letters, these letters have different exponents.

51. The **Reciprocal** of a quantity is unity divided by that quantity. Thus, the reciprocal of 2 is $\frac{1}{2}$, of a is $\frac{1}{a}$.

The reciprocal of $\frac{2}{3}$ is 1 divided by $\frac{2}{3}$, or $\frac{3}{2}$. Hence, *the reciprocal of a fraction is the fraction inverted.*

52. The same letter *accented* is often used to denote quantities which occupy similar positions in different equations or investigations.

Thus, a, a', a'', a''', represent four different quantities; read a, a prime, a second, a third, and so on.

EXAMPLES.

The following examples are intended to exercise the learner in the use and meaning of the signs.

Copy each example on the slate or blackboard, and then express it in common language.

Let the numerical values be found, on the supposition that $a=4$, $b=3$, $c=5$, $d=10$, $x=2$, and $y=6$.

1. $c+d-b$. . . Ans. 12.	5. $\frac{ay}{b}+\frac{cd}{x}$. . . . . Ans.
2. $4a-x$. . . . Ans. 14.	6. $3a^2+2cx-b^3$. Ans.
3. $-3ax$. . . Ans. -24.	7. $a(a+b)$. . . . . Ans.
4. $6a^2x$ . . . Ans. 192.	

REVIEW.—50. What are similar or like quantities? 51. The reciprocal of a quantity? 52. What the use of accented letters?

8. $a+b\times a-b$. . . . . Ans.
9. $(a+b)(a-b)$. . . . . Ans.
10. $x^2-3(a+x)(a-x)+2by$. . . . . Ans.
11. $\frac{2ax^2}{(a-x)^2}-6x\sqrt{a}$. . . . . Ans.
12. $3(a+c)(a-c)+3a^2-3c^2$. . . . . Ans.
13. $\frac{a^2-x^2}{a+x}+a-x$. . . . . Ans.

In the following, convert the words into algebraic symbols:

1. Three times a, plus b, minus four times c.
2. Five times a, divided by three times b.
3. a minus b, into three times c.
4. a, minus three times b into c.
5. a plus b, divided by three c.
6. a, plus b divided by three c.
7. a squared, minus three a into b, plus 5 times c into d squared.
8. x cubed minus b cubed, divided by x squared minus b squared.
9. Five a squared, into a plus b, into c minus d, minus three times x fourth power.
10. a squared plus b squared, divided by a plus b, squared.
11. The square root of a, minus the square root of x.
12. The square root of a minus x.

ANSWERS.

ADDITION.

53. Addition, in Algebra, is the process of finding the simplest expression for the sum of two or more algebraic quantities.

CASE I.

When the Quantities are similar, and have the same Sign.

1. James has 3 pockets, each containing apples: in the first he has 3 apples, in the second 4, and in third 5.

In order to find how many apples he has, suppose he proceeds to find their sum in the following manner:

3 *apples,*
4 *apples,*
5 *apples,*
———
12 *apples.*

But, instead of writing the word *apples,* suppose he should use the letter a, thus:

$$\begin{array}{r} 3a \\ 4a \\ 5a \\ \hline 12a \end{array}$$

It is evident that the sum of 3 times a, 4 times a, and 5 times a, is 12 times a, or $12a$, whatever a may represent.

2. In the same manner the sum of $-3a$, $-4a$, and $-5a$ would be $-12a$. Hence,

$$\begin{array}{r} -3a \\ -4a \\ -5a \\ \hline -12a \end{array}$$

TO ADD SIMILAR QUANTITIES WITH LIKE SIGNS,

Rule.—*Add together the coëfficients of the several quantities; to their sum prefix the common sign, and annex the common letter or letters.*

NOTE.—When a quantity has no coëfficient, 1 is understood; thus, $a=1a$.

REVIEW.—53. What is algebraic addition? When quantities are similar and have the same sign, how are they added together?

When several quantities are to be added together, is the result affected by the order in which they are taken?

EXAMPLES.

	(3)	(4)	(5)	(6)
	$3a$	$-6xy$	$2a^2$	$-3a^2b$
	$2a$	$-xy$	$3a^2$	$-4a^2b$
	a	$-4xy$	$5a^2$	$-5a^2b$
	$5a$	$-3xy$	$7a^2$	$-2a^2b$
Sum	$=11a$	$-14xy$	$17a^2$	$-14a^2b$

In the third example, suppose $a=2$, then $3a=3\times2=6$, $2a=2\times2=4$, $a=2$, $5a=5\times2=10$; their sum is $6+4+2+10=22$.

But the sum, 22, is more easily found from the algebraic sum, $11a$, for $11a=11\times2=22$.

In the fourth example, let $x=3$ and $y=2$; then,

$$\begin{aligned}-6xy&=-6\times3\times2=-36\\ -xy&=-\quad 3\times2=-\ 6\\ -4xy&=-4\times3\times2=-24\\ -3xy&=-3\times3\times2=-18\end{aligned}$$

the sum of their values $=-84$.

But this is more easily found thus: $-14xy=-14\times3\times2=-84$.

In the fifth example, let a represent three feet; then,

$2a^2=2aa=2\times3\times3=18$ square feet,
$3a^2=3aa=3\times3\times3=27$ " "
$5a^2=5aa=5\times3\times3=45$ " "
$7a^2=7aa=7\times3\times3=63$ " "

and their sum is 153 " "

Or the sum $=17a^2=17\times3\times3=153$ square feet.

NOTE.—Let the learner test the following examples numerically, by assigning values to the letters.

7. What is the sum of $3b$, $5b$, $7b$, and $9b$? Ans.
8. Of $2ab$, $5ab$, $8ab$, and $11ab$? Ans.
9. Of abc, $3abc$, $7abc$, and $12abc$? Ans.
10. Of $-by$, $-2by$, $-5by$, and $-8by$? Ans.

(11)	(12)	(13)
$3ay+7$	$8x-4y$	$3a^2-2ax$
$ay+8$	$5x-3y$	$5a^2-3ax$
$2ay+4$	$7x-6y$	$7a^2-5ax$
$5ay+6$	$6x-2y$	$4a^2-4ax$

CASE II.

54. *When Quantities are alike, but have Unlike Signs.*

1. James receives from one man 6 cents, from another 9, and from a third 10. He spends 4 cents for candy and 3 for apples: how much will he have left?

If the quantities he *received* be considered *positive*, those he *spent* may be considered *negative;* and the question is, to find the sum of $+6c$, $+9c$, $+10c$, $-4c$ and $-3c$, which may be written thus:

$$\begin{array}{r} +6c \\ +9c \\ +10c \\ -4c \\ -3c \\ \hline +18c \end{array}$$

It is evident the true result will be found by collecting the positive quantities into one sum, and the negative quantities into another, and taking their difference. It is thus found that he received $25c$, and spent $7c$, which left $18c$.

2. Suppose James should receive 5 cents, and spend 7 cents, what sum would he have left?

If we denote the $5c$ as positive, the $7c$ will be negative, and it is required to find the sum of $+5c$ and $-7c$.

In its present form it is evident that the question is impossible. But if we suppose that James had a certain sum of money before he received the $5c$, we may inquire what effect the operation had upon his money.

The answer obviously is, that his money was *diminished* 2 cents; this would be indicated by the sum of $+5c$ and $-7c$, being $-2c$.

Hence, we say that the sum of a positive and negative quantity is equal to the *difference* between the two; the object being to find what the *united effect* of the two will be upon some third quantity.

This may be further illustrated by the following example:

3. A merchant has a certain capital; during the year it is *increased* by $3a$ and $8a$ \$'s, and *diminished* by $2a$ and $5a$ \$'s: how much will it be increased or diminished at the close of the year?

If we call the *gains* positive, the *losses* will be negative. The sum of $+3a$, $+8a$, $-2a$, and $-5a$, is $+11a-7a=+4a$.

Hence, we say that the merchant's capital will be *increased* by $4a$ \$'s, which is the same as to increase it by $3a$ and $8a$ \$'s, and then diminish it by $2a$ and $5a$ \$'s.