ECLECTIC EDUCATION SERIES

Ray's New Primary Arithmetic

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ECLECTIC EDUCATIONAL SERIES.

RAY'S

NEW PRIMARY

ARITHMETIC

FOR

YOUNG LEARNERS.

VAN ANTWERP, BRAGG & CO.

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ECLECTIC PRESS
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PREFACE.

THE remarkable and long continued popularity of *Ray's Arithmetics* has induced the publishers to present them to the public in a revised form, as *Ray's New Arithmetics.*

The objects of the revision have been:

(1.) To present the books in improved type, with better arrangement, and in a more pleasing outward dress.

(2.) To introduce such new features as will adapt the series more perfectly to the present methods of instruction.

The friends of the series can best judge what success in attaining these objects has been made.

The publishers hereby express their gratitude to many prominent educators who have contributed to this revision, and only regret that their number prevents the mention of names.

CINCINNATI, *April,* 1877.

SUGGESTIONS TO TEACHERS.

IN beginning the study of Arithmetic, the first step for pupils to learn is to count readily. This is not mastered without much practice in counting *objects*. Movable objects are better for exercises in counting than pictures. Some objects of this kind should always be kept in the school-room,—such as marbles, beans, kernels of corn, or pebbles.

The second step is to combine numbers. To master the different combinations to 20, the pupils should first be taught to write the tables corresponding with those in the book, either upon their slates or on the blackboard, during the recitation. This will prevent counting upon the fingers, a habit difficult to overcome when once acquired.

As the abstract exercises in this book, up to 20, are exhaustive in Addition and Subtraction, and as complete in Multiplication and Division as possible in order to secure variety, it would be well to prepare additional concrete examples from day to day to correspond with the very full abstract exercises. An excellent practice is to require each pupil to bring two or more concrete examples of his own to each recitation.

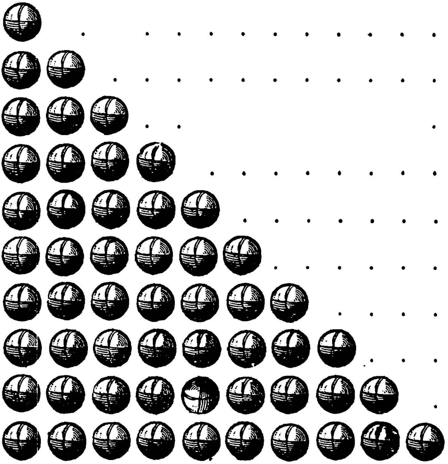
Teach one thing at a time, and teach it thoroughly.

PRIMARY ARITHMETIC

NUMBERS AND FIGURES.

LESSON I.

NOTE.—This lesson is intended to suggest how to teach the child to count objects and to express their number by figures. It comprises the first ten numbers. The pupil points to each ball and says, *one; one, two; one, two, three*, etc. Then the teacher directs him to write the figure for *one*, for *two*, etc.



. . . .	one, .	1,	*1.*
. . . .	two, .	2,	*2.*
. . . .	three, .	3,	*3.*
. . . .	four, .	4,	*4.*
. . . .	five, .	5,	*5.*
. . . .	six, .	6,	*6.*
. . . . . .	seven, .	7,	*7.*
. . . . .	eight, .	8,	*8.*
. .	nine, .	9,	*9.*
.	ten, .	10,	*10.*

LESSON II.

NOTE.—In this lesson the numbering of objects is extended to 40. A single column or less may constitute one exercise, as may seem best to the teacher.

Eleven	11	Twenty-one	21	Thirty-one	31
Twelve	12	Twenty-two	22	Thirty-two	32
Thirteen	13	Twenty-three	23	Thirty-three	33
Fourteen	14	Twenty-four	24	Thirty-four	34
Fifteen	15	Twenty-five	25	Thirty-five	35
Sixteen	16	Twenty-six	26	Thirty-six	36
Seventeen	17	Twenty-seven	27	Thirty-seven	37
Eighteen	18	Twenty-eight	28	Thirty-eight	38
Nineteen	19	Twenty-nine	29	Thirty-nine	39
TWENTY	20	THIRTY	30	FORTY	40

LESSON III.

NOTE.—In this lesson the numbering of objects is extended to 70. Each exercise should include a review of the preceding ones.

Forty-one	41	Fifty-one	51	Sixty-one	61
Forty-two	42	Fifty-two	52	Sixty-two	62
Forty-three	43	Fifty-three	53	Sixty-three	63
Forty-four	44	Fifty-four	54	Sixty-four	64
Forty-five	45	Fifty-five	55	Sixty-five	65
Forty-six	46	Fifty-six	56	Sixty-six	66
Forty-seven	47	Fifty-seven	57	Sixty-seven	67
Forty-eight	48	Fifty-eight	58	Sixty-eight	68
Forty-nine	49	Fifty-nine	59	Sixty-nine	69
FIFTY	50	SIXTY	60	SEVENTY	70

LESSON IV.

Seventy-one	71	Eighty-one	81	Ninety-one	91
Seventy-two	72	Eighty-two	82	Ninety-two	92
Seventy-three	73	Eighty-three	83	Ninety-three	93
Seventy-four	74	Eighty-four	84	Ninety-four	94
Seventy-five	75	Eighty-five	85	Ninety-five	95
Seventy-six	76	Eighty-six	86	Ninety-six	96
Seventy-seven	77	Eighty-seven	87	Ninety-seven	97
Seventy-eight	78	Eighty-eight	88	Ninety-eight	98
Seventy-nine	79	Eighty-nine	89	Ninety-nine	99
EIGHTY	80	NINETY	90	ONE HUNDRED	100

LESSON V.

NOTE.—Pupils should be taught to *read* figures readily from 1 to 100. The figures should be copied on the blackboard.

NUMBERS TO BE READ.

0	23	50	61	71	80	78	59
1	32	15	26	27	18	87	95
10	33	51	62	72	81	88	69
11	4	25	36	37	28	9	96
2	40	52	63	73	82	90	79
20	14	35	46	47	38	19	97
12	41	53	64	74	83	91	89
21	24	45	56	57	48	29	98
22	42	54	65	75	84	92	99
3	34	55	66	67	58	39	100
30	43	6	7	76	85	93	
13	44	60	70	77	68	49	
31	5	16	17	8	86	94	

LESSON VI.

NOTE.—The pupils must be thoroughly exercised in *writing* numbers. One or more pupils at a time may be sent to the blackboard, or the work may be done at their seats with pencil and slate.

NUMBERS TO BE WRITTEN.

1. Naught; one, ten; two, twenty; three, thirty; four, forty; five, fifty; six, sixty; seven, seventy; eight, eighty; nine, ninety.
2. Eleven; twelve, twenty-one; thirteen, thirty-one; fourteen, forty-one; fifteen, fifty-one; sixteen, sixty-one; seventeen, seventy-one; eighteen, eighty-one; nineteen, ninety-one.
3. Twenty-two; twenty-three, thirty-two; twenty-four, forty-two; twenty-five, fifty-two; twenty-six, sixty-two; twenty-seven, seventy-two; twenty-eight, eighty-two; twenty-nine, ninety-two.
4. Thirty-three; thirty-four, forty-three; thirty-five, fifty-three; thirty-six, sixty-three; thirty-seven, seventy-three; thirty-eight, eighty-three; thirty-nine, ninety-three.
5. Forty-four; forty-five, fifty-four; forty-six, sixty-four; forty-seven, seventy-four; forty-eight, eighty-four; forty-nine, ninety-four.
6. Fifty-five; fifty-six, sixty-five; fifty-seven, seventy-five; fifty-eight, eighty-five; fifty-nine, ninety-five.
7. Sixty-six; sixty-seven, seventy-six; sixty-eight, eighty-six; sixty-nine, ninety-six.
8. Seventy-seven; seventy-eight, eighty-seven; seventy-nine, ninety-seven.
9. Eighty-eight; eighty-nine, ninety-eight.
10. Ninety-nine. One hundred.

ORAL EXERCISES.

LESSON VII.

NOTE.—These exercises are intended for use with the Numeral Frame or with counters of some kind,—marbles, pebbles, kernels of corn, beans, or bits of pasteboard. The objects should be arranged in distinct groups, to represent each number indicated.

1. How many counters have we here? (/)

2. How many are 1 and 1? One taken away from 2 leaves how many? How many ones in 2? How many are two times 1?

3. How many are 2 and 1? How many are 1 and 1 and 1? How many are three times 1?

4. One taken away from 3 leaves how many? Two taken away from 3 leaves how many? How many ones in 3?

5. How many are 3 and 1? How many are 2 and 2? How many are 1 and 1 and 1 and 1? How many are four times 1? How many are two times 2?

6. One taken from 4 leaves how many? Two from 4

leaves how many? Three from 4 leaves how many? How many ones in 4? How many twos in 4?

7. How many are 4 and 1? How many are 3 and 2? How many are 1 and 1 and 1 and 1 and 1? How many are five times 1?

8. One from 5 leaves how many? Two from 5 leaves how many? Three from 5 leaves how many? Four from 5 leaves how many? How many ones in 5?

LESSON VIII.

1. How many are 5 and 1? How many are 4 and 2? How many are 3 and 3? How many are six times 1? How many are three times 2? How many are two times 3?

2. One from 6 leaves how many? Two from 6? Three from 6? Four from 6? Five from 6? How many ones in 6? How many twos in 6? How many threes in 6?

3. How many are 6 and 1? How many are 5 and 2? How many are 4 and 3? How many are 3 and 4? How many are seven times 1?

4. One from 7 leaves how many? Two from 7? Three from 7? Four from 7? Five from 7? Six from 7? How many ones in 7?

5. How many are 7 and 1? How many are 6 and 2? How many are 5 and 3? How many are 4 and 4? How many are 3 and 5? How many are 2 and 6?

6. How many are eight times 1? How many are four times 2? How many are two times 4?

7. One from 8 leaves how many? Two from 8? Three from 8? Four from 8? Five from 8? Six from 8? Seven from 8?

8. How many ones in 8? How many twos in 8? How many fours in 8?

LESSON IX.

1. How many are 8 and 1? How many are 7 and 2? How many are 6 and 3? How many are 5 and 4? How many are 4 and 5? How many are 3 and 6? How many are 2 and 7?

2. How many are nine times 1? How many are three times 3?

3. One from 9 leaves how many? Two from 9? Three from 9? Four from 9? Five from 9? Six from 9? Seven from 9? Eight from 9?

4. How many ones in 9? How many threes in 9?

5. How many are 9 and 1? How many are 8 and 2? How many are 7 and 3? How many are 6 and 4? How many are 5 and 5?

6. How many are 2 and 8? How many are 3 and 7? How many are 4 and 6?

7. How many are ten times 1? How many are five times 2? How many are two times 5?

8. One from 10 leaves how many? Two from 10? Three from 10? Four from 10? Five from 10? Six from 10? Seven from 10? Eight from 10? Nine from 10?

9. How many ones in 10? How many twos in 10? How many fives in 10?

LESSON X.

In the picture how many birds are sitting on the bush? How many in the flock that seems to be lighting? There are two distant flocks flying: how many birds in each flock?

1. How many birds are two birds and five birds? How many birds are seven birds and four birds?

2. How many are 2 and 5? 7 and 4?

3. How many birds are two birds and four birds? How many are five birds and seven birds?

4. How many are 2 and 4? 5 and 7?

5. There are three flowers on one branch and three on another: how many flowers on both branches?

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Ray's New Intellectual Arithmetic

LESSON XLII.

1. A man, having $10\frac{2}{7}$ acres of land, divided it equally among his 6 children: how much did each receive?

SOLUTION.—$10\frac{2}{7}=\frac{72}{7}$. Each received $\frac{1}{6}$ of $\frac{72}{7}$ acres, which is $\frac{12}{7}$, or $1\frac{5}{7}$ acres.

2. If $2\frac{4}{5}$ be divided by 7, what will be the result?
3. How many times is 6 contained in $3\frac{3}{5}$?
4. How many times is 9 contained in $6\frac{3}{4}$?
5. Divide $8\frac{3}{4}$ by 5. $7\frac{2}{4}$ by 10.
6. Divide $4\frac{5}{7}$ by 11. $8\frac{4}{7}$ by 12.
7. If $1\frac{1}{2}$ yards of ribbon cost 6 cents, what will 1 yard cost?

SOLUTION.—$1\frac{1}{2}=\frac{3}{2}$. $\frac{1}{2}$ a yard cost $\frac{1}{3}$ of 6 cents = 2 cents; then, 1 yard cost 2 times 2 cents = 4 cents.

8. If $1\frac{1}{3}$ yards of cloth cost $4, what will 1 yard cost?
9. If a man travel 9 miles in $1\frac{2}{7}$ hours, how far will he travel in 1 hour?
10. A watch was sold for $18, which equaled $1\frac{1}{5}$ of what it cost me: how much did it cost?
11. A grocer sold a lot of flour for $25, which was $1\frac{1}{4}$ times the cost: what did it cost? How much did he gain?
12. If a man pays $6 for $1\frac{1}{3}$ yards of cloth, what is the cost of 1 yard?
13. If a man receives $10 for $2\frac{2}{3}$ days work, how much is that a day?
14. If a man receives $12 for $6\frac{2}{5}$ days work, how much is that a day?
15. How many are 9 divided by $3\frac{3}{4}$?
16. How many are 10 divided by $2\frac{1}{7}$?
17. How many are 11 divided by $4\frac{8}{9}$?

LESSON XLIII.

1. If a yard of cloth cost $\$\frac{2}{3}$, how many yards will cost $\$4\frac{6}{7}$?

SOLUTION.—$4\frac{6}{7}=\frac{34}{7}$. As many yards as $\frac{2}{3}$ are contained times in $\frac{34}{7}$, which are $\frac{51}{7}=7\frac{2}{7}$.

2. When a bushel of corn costs $\$\frac{1}{2}$, how many bushels can you buy for $\$1\frac{1}{2}$?

3. I distributed $2\frac{2}{3}$ bushels of wheat among a number of poor persons, giving to each $\frac{2}{3}$ of a bushel: how many persons were there?

4. At $\$\frac{1}{4}$ a yard, how many yards of alpaca can be purchased for $\$3\frac{3}{4}$?

5. At $\$\frac{3}{4}$ a yard, how many yards of cloth can be purchased for $\$3\frac{1}{4}$?

6. If an apple cost $\frac{3}{4}$ of a cent, how many apples can be purchased for $3\frac{3}{4}$ cents? For $5\frac{1}{4}$ cents?

7. If a yard of cloth cost $\$\frac{2}{3}$, how many yards can you purchase for $\$4\frac{1}{3}$?

8. How often is $1\frac{1}{2}$ contained in $\frac{3}{4}$? In $\frac{4}{5}$? In $2\frac{3}{4}$?

9. How often is $2\frac{1}{4}$ contained in $\frac{5}{6}$? In $\frac{5}{7}$? In $3\frac{7}{8}$?

10. How often is $3\frac{1}{5}$ contained in $\frac{3}{8}$? In $\frac{3}{7}$? In $5\frac{3}{5}$?

11. $5\frac{1}{3}$ is $\frac{1}{2}$ of what number? $\frac{1}{5}$ of what number?

12. $7\frac{3}{4}$ is $\frac{1}{3}$ of what number? $\frac{1}{7}$ of what number?

13. $9\frac{2}{3}$ are $\frac{5}{8}$ of what number? $\frac{5}{6}$ of what number?

14. $4\frac{2}{3}$ are $\frac{2}{5}$ of what number? $\frac{5}{6}$ of what number?

15. $3\frac{2}{3}$ are $\frac{3}{4}$ of what number? $\frac{3}{5}$ of what number?

16. How often is $\frac{1}{6}$ contained in $3\frac{5}{6}$? In $5\frac{1}{6}$? In $4\frac{4}{6}$?

17. How often are $\frac{3}{5}$ contained in $2\frac{2}{5}$? In $4\frac{3}{5}$? In $6\frac{1}{5}$?

18. How often are $\frac{3}{7}$ contained in $3\frac{2}{5}$? In $4\frac{2}{3}$? In $7\frac{3}{4}$?

19. How often are $\frac{5}{8}$ contained in $4\frac{3}{4}$? In $5\frac{3}{5}$? In $8\frac{4}{7}$?

20. How often are $\frac{2}{3}$ contained in $2\frac{3}{10}$? In $6\frac{9}{11}$? In $9\frac{5}{12}$? In $10\frac{2}{3}$?

21. At $\$\frac{2}{5}$ a gallon, how many gallons of vinegar can you buy for $\$2\frac{2}{5}$? For $\$4\frac{1}{5}$?

22. One bushel of rye is worth $\frac{3}{4}$ of a bushel of wheat: how many bushels of rye can be bought with $4\frac{1}{2}$ bushels of wheat? With $8\frac{1}{4}$ bushels?

LESSON XLIV.

The examples in this lesson are to be solved by using the following tables, where applicable:

I.—Fractional Parts of 12.

$2 = \frac{1}{6}$	$6 = \frac{1}{2}$.
$3 = \frac{1}{4}$	$8 = \frac{2}{3}$.
$4 = \frac{1}{3}$	$9 = \frac{3}{4}$.

$10 = \frac{5}{6}$.

II.—Fractional Parts of 100.

$12\frac{1}{2} = \frac{1}{8}$	$37\frac{1}{2} = \frac{3}{8}$.
$16\frac{2}{3} = \frac{1}{6}$	$50 = \frac{1}{2}$.
$20 = \frac{1}{5}$	$62\frac{1}{2} = \frac{5}{8}$.
$25 = \frac{1}{4}$	$66\frac{2}{3} = \frac{2}{3}$.
$33\frac{1}{3} = \frac{1}{3}$	$75 = \frac{3}{4}$.

$87\frac{1}{2} = \frac{7}{8}$.

1. Bought $\frac{3}{4}$ of a dozen shirts, at \$24 a dozen: what did they cost?

Solution.—They cost $\frac{3}{4}$ of $\$24 = \$18$.

2. Bought $\frac{2}{3}$ of a dozen linen collars, at \$3 a dozen: what did they cost?

3. Bought $\frac{5}{6}$ of a dozen handkerchiefs, at \$4 a dozen: how much did they cost?

4. A grocer bought $6\frac{1}{2}$ dozen eggs, for 16 cents a dozen: how much did they cost?

5. Bought $1\frac{2}{3}$ dozen pairs of hose, for $\$2\frac{2}{5}$ a dozen: how much did they cost? What did each pair cost?

SOLUTION.—$1\frac{2}{3}=\frac{5}{3}$; $2\frac{2}{5}=\frac{12}{5}$. They cost $\frac{5}{3}$ of $\$\frac{12}{5}=\$4$. Each pair cost $\frac{1}{12}$ of $\$\frac{12}{5}=\$\frac{1}{5}$.

6. Bought $2\frac{1}{4}$ dozen copy-books, for $\$1\frac{1}{5}$ a dozen: how much did they cost? What was the cost of each book?

7. A merchant bought $6\frac{1}{2}$ dozen knives, for $\$1\frac{4}{5}$ a dozen: what did they cost? What did 1 knife cost?

8. Paid \$5 a set, or $\frac{1}{2}$ dozen, for $2\frac{1}{2}$ dozen spoons: what did they cost?

9. Bought $4\frac{1}{6}$ dozen spelling-books, at $\$2\frac{1}{4}$ a dozen: how much did they cost? What did 1 book cost?

10. A man bought $2\frac{1}{4}$ dozen handkerchiefs, for $\$6\frac{3}{4}$: how much was that apiece?

SOLUTION.—$2\frac{1}{4}=\frac{9}{4}$; $6\frac{3}{4}=\frac{27}{4}$. He bought $\frac{1}{4}$ of a dozen, or 3, handkerchiefs, for $\frac{1}{9}$ of $\$\frac{27}{4}=\$\frac{3}{4}$; then, 1 handkerchief cost $\frac{1}{3}$ of $\$\frac{3}{4}=\$\frac{1}{4}$.

11. A merchant paid $\$3\frac{1}{10}$ for $7\frac{3}{4}$ dozen pairs of damaged hose, and sold them for $\$\frac{1}{10}$ a pair: how much did he gain on each pair?

12. A merchant paid \$15 for $2\frac{1}{2}$ dozen silk handkerchiefs, and sold them for $\$\frac{3}{5}$ apiece: how much did he gain on each handkerchief? How much on the whole lot?

13. Paid $\$18\frac{3}{4}$ for $6\frac{1}{4}$ dozen knives, and sold them for $\$2\frac{1}{10}$ a set, or $\frac{1}{2}$ doz.: how much did I gain?

14. What will 16 pounds of soap cost, at $12\frac{1}{2}$ cents a pound?

SOLUTION.—$12\frac{1}{2}$ cents $=\$\frac{1}{8}$; then. 16 pounds will cost 16 times $\$\frac{1}{8}=\frac{16}{8}$, or \$2.

15. What will 12 pounds of prunes cost, at $16\frac{2}{3}$ cents a pound?

16. What will 24 yards of alpaca cost, at $37\frac{1}{2}$ cents a yard?

17. What will 16 yards of flannel cost, at $62\frac{1}{2}$ cents a yard?

18. What will 15 pounds of coffee cost, at $33\frac{1}{3}$ cents a pound?

19. What will 27 yards of flannel cost, at $66\frac{2}{3}$ cents a yard?

20. What will 15 yards of cloth cost, at $1.66\frac{2}{3}$ a yard?

21. Paid $12 for coffee, at $33\frac{1}{3}$ cents a pound: how many pounds did I buy?

SOLUTION.—$33\frac{1}{3}$ cents = $\frac{1}{3}$ dollar. I bought as many pounds as $\frac{1}{3}$ is contained times in 12, which are 36.

22. Paid $1\frac{1}{4}$ dollars for eggs, at $12\frac{1}{2}$ cents a dozen: how many dozen did I buy?

23. Paid $7\frac{1}{2}$ dollars for flannel, at $62\frac{1}{2}$ cents a yard: how many yards did I buy?

24. Paid $8 for flannel, at $66\frac{2}{3}$ cents a yard: how many yards did I buy?

25. Multiply 32 by $12\frac{1}{2}$.

SOLUTION.—$12\frac{1}{2} = \frac{1}{8}$ of 100; then, $32 \times 12\frac{1}{2} = 32 \div 8 \times 100 = 400$.

26. Multiply 18 by 50.	40 by $62\frac{1}{2}$.	68 by 75.
27. Multiply 48 by 75.	24 by $37\frac{1}{2}$.	51 by $33\frac{1}{3}$.
28. Multiply 39 by $66\frac{2}{3}$.	64 by $87\frac{1}{2}$.	96 by $62\frac{1}{2}$.

29. Divide 150 by $12\frac{1}{2}$.

SOLUTION.—$150 \div 12\frac{1}{2} = 150 \times 8 \div 100 = 12$.

30. Divide 200 by $16\frac{2}{3}$.	560 by 20.	250 by 25.
31. Divide 350 by $37\frac{1}{2}$.	600 by 50.	750 by $62\frac{1}{2}$.

REVIEW.

LESSON XLV.

1. William had 23 cents: Thomas gave him 8 cents more, George 6, James 5, and David 7; he gave 15 cents for a book: how many cents had he left?

2. A grocer paid $12 for sugar, $9 for coffee, $5 for tea, $7 for flour, and had $10 left: how many dollars had he at first?

3. A boy has 11 cents: his father gives him 9 cents, his mother 6, and his sister enough more to make 34: how many cents does his sister give him?

4. Five men bought a horse for $42: the first gave $13; the second, $7; the third, $5; and the fourth, $9: how many dollars did the fifth give?

5. A man purchased 8 sheep, at $4 a head; 5 barrels of flour, at $3 a barrel; 4 yards of cloth, at $3 a yard; and 5 ounces of opium, at $1 an ounce: how much did he spend?

6. A boy lost 25 cents: after finding 15 cents, he had 25: how many cents had he at first?

7. A man owed a debt of $28, and paid all but $9: how much did he pay?

8. Borrowed $56: at one time I paid $23; at another, all but $7: how much did I pay the last time?

9. James borrowed 37 cents: at one time he paid 5 cents, at another 8, and the third time, all but 15: how many cents did he pay the third time?

10. A farmer sold 1 cow, at $18, and 5 pigs, at $3 each, receiving in payment 3 sheep, at $3 each, and the rest in money: how much money did he receive?

11. A farmer sold 12 barrels of cider, at $3 a barrel: he then purchased 5 barrels of salt, at $3 a barrel, and some sugar, for $8: how many dollars had he left?

12. A merchant purchased 13 hats, at $4 each; 5 pairs of shoes, at $2 a pair; and an umbrella, for $7: what must he sell the whole for to gain $9?

13. If 2 barrels of flour cost $12, what will 7 barrels cost? 5 barrels?

14. If 3 barrels of cider cost $12, what will 4 barrels cost? 9 barrels?

15. If 4 yards of cloth cost $28, what will 7 yards cost?

16. If 5 tons of hay cost $35, what will 8 tons cost?

17. If 7 apples cost 28 cents, what will 3 apples cost?

18. If 8 oranges are worth 24 apples, how many apples are 3 oranges worth?

19. If 2 pounds of cheese cost 36 cents, what will 3 pounds cost?

20. If 8 yards of cloth cost $56, what will 7 yards cost?

21. If 9 yards of calico cost 72 cents, what will 6 yards cost? 8 yards? 10 yards?

22. A walks 5 miles, while B walks 3: when A has gone 35 miles, how far has B gone?

23. Joseph and his father are husking corn: the father can husk 7 rows while Joseph husks 3: how many rows will Joseph husk while his father husks 42?

24. Charles can earn $9 while Mary earns $4: how many dollars will Charles earn while Mary earns $28.

25. If 6 horses eat 12 bushels of oats in a week, how many bushels will 10 horses eat in the same time?

26. If five horses eat 16 bushels in 2 weeks, how long would it take them to eat 56 bushels?

27. If 6 apples are worth 18 cents, how many apples must be given for 5 oranges, worth 6 cents each?

28. How many horses can eat in 9 days the same amount of hay that 12 horses eat in 6 days?

LESSON XLVI.

1. If 4 yards of cloth cost $16, what will 5 yards cost? 9 yards?

2. What are $\frac{8}{9}$ of 72? $\frac{9}{8}$ of 72?

3. If you had 64 cents, how many oranges could you buy, at 8 cents each?

4. Ninety-six is how many times 6?

5. James had 48 chestnuts: he gave $\frac{1}{2}$ of them to his brother, and $\frac{1}{3}$ to his sister: how many had he left?

6. Nine times 9 are how many times 12?

7. In $8\frac{5}{9}$ how many ninths? In $9\frac{1}{9}$?

8. Reduce $\frac{48}{120}$, $\frac{54}{189}$, $\frac{240}{288}$, to their lowest terms.

9. Reduce $\frac{3}{9}$, $\frac{4}{16}$, $\frac{17}{72}$, to a least common denominator.

10. A farmer planted $4\frac{1}{2}$ acres in potatoes, $20\frac{3}{4}$ acres in wheat, and $24\frac{7}{8}$ acres in oats: how many acres did he plant?

11. From $9\frac{3}{8}$ take $5\frac{2}{3}$.

12. A man having 84 miles to travel, went $\frac{1}{4}$ of the distance the first day, $\frac{1}{3}$ the second, and the rest the third day: what part did he travel the last day, and how far?

13. What are 9 times $\frac{7}{18}$?

14. What are $\frac{7}{11}$ of 12?

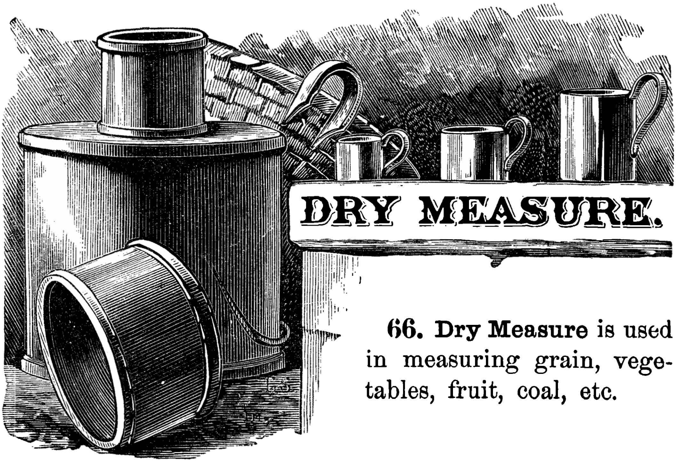
15. If 4 yards of cloth cost $15, what will 7 yards cost?

16. How many are 7 times $7\frac{5}{7}$?

17. Four times $6\frac{3}{7}$ are how many times 7?

ECLECTIC EDUCATION SERIES

Ray's Elementary Arithmetic



66. Dry Measure is used in measuring grain, vegetables, fruit, coal, etc.

TABLE.

2 pints (pt)	make	1 quart,	marked	qt.
8 quarts	"	1 peck,	"	pk.
4 pecks	"	1 bushel,	"	bu.

NOTES.—1. The standard of dry measure is the bushel, which is a cylindrical measure $18\frac{1}{2}$ inches in diameter, 8 inches deep, and containing $2150\frac{2}{5}$ cubic inches.

2. The English quarter contains 8 bushels of 70 pounds each, and is used in measuring grain.

3. The chaldron, used for measuring coal, is employed in England and some of the United States. It contains 36 bushels.

MENTAL EXERCISES.

EXAMPLE.—How many pints are there in 4 quarts?

SOLUTION.—Since in 1 quart there are 2 pints, in 4 quarts there are 4 times 2 pints, which is 8 pints.

CONCLUSION.—Therefore, in 4 quarts there are 8 pints.

1. How many pints in 3 quarts? In 5 qt.? In 6 qt.? In 7 qt.?
2. How many quarts in 2 pecks? In 3 pk.? In 4 pk.? In 5 pk.?
3. How many pecks in 2 bushels? In 4 bu.? In 3 bu.? In 5 bu.?
4. How many pecks in 6 bushels? In 9 bu.? In 8 bu.? In 10 bu.?

EXAMPLE.—How many quarts are there in 6 pints?

SOLUTION.—Since 2 pints equal 1 quart, 6 pints equal as many quarts as 2 is contained times in 6, which are 3.

CONCLUSION.—Therefore, in 6 pints there are 3 quarts.

5. How many quarts in 4 pints? In 8 pt.? In 12 pt.? In 16 pt.?
6. How many pecks in 8 quarts? In 32 qt.? In 64 qt.? In 24 qt.?
7. How many bushels in 8 pecks? In 32 pk.? In 16 pk.?
8. How many bushels in 12 pk.? In 20 pk.? In 64 qt.?

MODEL SOLUTIONS.

EXAMPLE.—Reduce 177 pints to higher denominations.

SOLUTION.—Since 2 pt. make 1 qt., 177 pt. make as many qt. as 2 is contained times in 177, = 88 qt. and 1 pt. remaining.

Since 8 qt. make 1 pk., 88 qt. make as many pk. as 8 is contained times in 88, = 11 pk.

Since 4 pk. make 1 bu., 11 pk. make as many bu. as 4 is contained times in 11, = 2 bu. and 3 pk. remaining.

OPERATION.

```
2)177 pt.
 8)88 qt. 1 pt.
  4)11 pk.
     2 bu. 3 pk.
```

CONCLUSION.—Therefore, in 177 pt. there are 2 bu. 3 pk. 1 pt.

EXAMPLE.—Reduce 4 bu. 1 pk. 7 qt. 1 pt. to pints.

OPERATION.

bu.	pk.	qt.	pt.
4	1	7	1

$$
\begin{array}{rl}
4 & \\
\hline
16 & \text{pk.} \\
1 & \\
\hline
17 & \text{pk.} \\
8 & \\
\hline
136 & \text{qt.} \\
7 & \\
\hline
143 & \text{qt.} \\
2 & \\
\hline
286 & \text{pt.} \\
1 & \\
\hline
287 & \text{pt.}
\end{array}
$$

SOLUTION.—Since in 1 bu. there are 4 pk., in 4 bu. there are 4 times 4 pk., which are 16 pk.; 16 pk. + 1 pk. = 17 pk.

Since in 1 pk. there are 8 qt., in 17 pk. there are 17 times 8 qt., which are 136 qt.; 136 qt. + 7 qt. = 143 qt.

Since in 1 qt. there are 2 pt., in 143 qt. there are 143 times 2 pt., which are 286 pt.; 286 pt. + 1 pt. = 287 pt.

CONCLUSION.—Therefore, in 4 bu. 1 pk. 7 qt. 1 pt. there are 287 pt.

Hence we deduce the following rules for Reduction:

From Higher to Lower Denominations.—*Multiply the highest denomination given, by that number which it takes of the next lower denomination to make one of this higher; add to the product the number, if any, of the next lower denomination.*

Proceed in like manner with the result obtained, until the whole is reduced to the required denomination.

From Lower to Higher Denominations.—*Divide the given quantity by the number of units of its own denomination which make one of the next higher.*

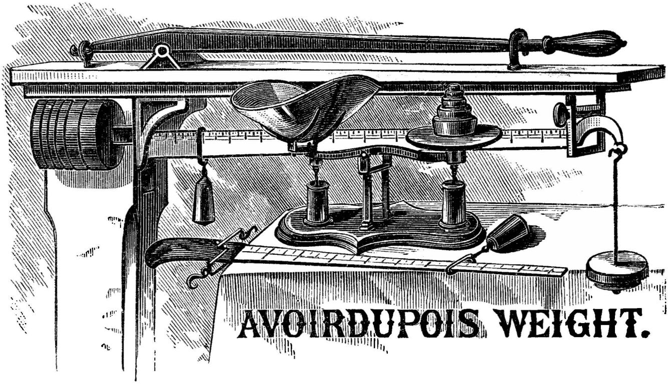
Proceed in like manner with the quotient thus obtained, till the whole is reduced to the required denomination.

The last quotient, with the several remainders, if any, annexed, will be the answer.

PROOF.—Reverse the operation; that is, reduce the answer back to the denomination from which it was derived.

EXAMPLES.

1. Reduce 2 bu. to pints.
2. Reduce 172 pt. to bushels.
3. Reduce 5 bu. 1 pk. to pints.
4. Reduce 408 pt. to bushels.
5. Reduce 1 bu. 1 pk. 1 qt. to quarts.
6. Reduce 18 bu. 3 pk. to pints.
7. Reduce 1803 pt. to bushels.
8. Reduce 12 bu. 1 pk. 3 qt. to pints.
9. Reduce 21132 qt. to bushels.
10. Reduce 24188 pt. to pecks.
11. Bought 10 bushels of plums at 5 cents a quart: what did they cost?
12. Bought 8 bu. 3 pk. of blackberries at 8 cents a quart, and sold them for 10 cents a quart: how much did I gain?
13. A man sold 108 bu. 3 qt. of corn at the rate of one cent a pint: what did it bring?
14. How many bushels of wheat can be purchased for $17.30, at 2 cents a pint?
15. In fifteen cars of wheat containing 300 bushels each, how many quarts?
16. A dealer bought a load of apples for $10.50. The price paid per bushel was 42 cents. How much did he gain by selling all at 20 cents a peck?
17. In 100000 pt. how many bushels?
18. How many bags, each containing 2 bu. 2 pk., will be needed to contain 62 bu. 2 pk.?
19. A dealer bought 40 barrels of apples, each containing 2 bu. 1 pk., at $2 a barrel. He retailed them at 30 ct. a peck: how much did he gain?
20. How many pecks of potatoes in eighty-seven pints?



67. Avoirdupois Weight is used in weighing ordinary articles that are bought and sold by weight, such as groceries, metals, drugs by wholesale, etc.

TABLE.

16 ounces (oz.)	make	1 pound,	marked	lb.
100 pounds	"	1 hundred-weight,	"	cwt.
20 cwt., or 2000 lb.,	"	1 ton,	"	T.

NOTES.—1. The *standard* avoirdupois pound of the United States is determined from the Troy pound, and contains 7000 gr. Troy.

2. At the Custom-house and in some trades, 2240 pounds are considered a ton.

MENTAL EXERCISES.

1. How many ounces in one half a pound?

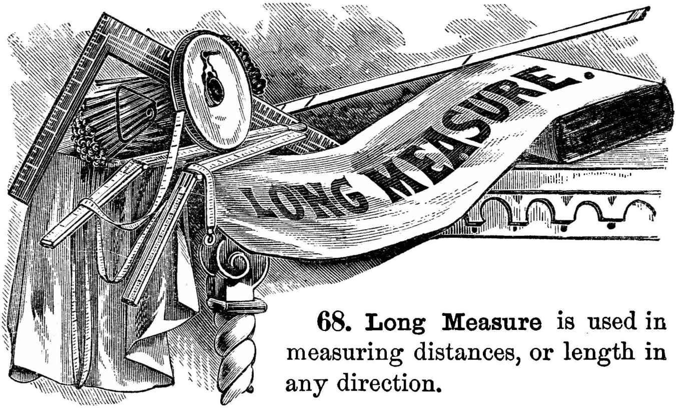
2. How many ounces in one quarter of a pound? How many in two quarters? How many in three quarters?

3. How many ounces in 3 pounds? In 2 lb.? In 4 lb.?

4. How many pounds in 32 ounces? In 96 oz.? In 80 oz.? In 48 oz.?

EXAMPLES.

1. Reduce 1 T. to ounces.
2. Reduce 32000 oz. to tons.
3. Reduce 325 lb. to ounces.
4. Reduce 1000224 oz. to tons.
5. Reduce 316450 ounces to denominations of higher orders.
6. Reduce 14 T. 1512 lb. 14 oz. to ounces.
7. Reduce 360 cwt. 5 lb. to ounces.
8. Reduce 8374160 oz. to tons.
9. Reduce 14 lb. 10 oz. to ounces.
10. Reduce 17460 lb. to tons.
11. Bought 2345 lb. of hay, at one cent a pound: what did it cost?
12. What is the cost of 3014 lb. of sugar, at 15 cents a pound?
13. What is the cost of 20094 lb. of iron, at 4 cents a pound?
14. A man bought 15 cwt. of lead, at 10 cents a pound, and paid for it with sugar, at 15 cents a pound: how much sugar did it take?
15. At 25 cents a pound, what quantity of grapes can be bought for $450?
16. How many pounds will 36 books weigh, if the weight of each book is 20 oz.?
17. The United States Postal Department charges 1 cent an ounce for all matter of the "third class." What would be the postage on a parcel of this nature weighing 3 lb. 3 oz.?
18. Reduce $17\frac{1}{2}$ T. to ounces.
19. Reduce 2 T. 12 cwt. 8 lb. 4 oz. to ounces.
20. In one load of hay there were 1480 pounds; in another, 1520 pounds. How much in both?



68. Long Measure is used in measuring distances, or length in any direction.

TABLE.

12	inches (in.)	make	1 foot,	marked	ft.
3	feet	"	1 yard,	"	yd.
$5\frac{1}{2}$	yards	"	1 rod,	"	rd.
320	rods	"	1 mile,	"	mi.

NOTES.—1. *The yard* is the standard unit of length. The standard yard for the United States is kept at Washington. A copy is deposited at the capital of each State.

2. The yard and its subdivisions are used in measuring cloth, ribbon, lace, etc.

MENTAL EXERCISES.

1. How many inches in 2 feet? In 4 ft.? In 3 ft.? In 5 ft.?

2 How many feet in 48 inches? In 72 in.? In 120 in.? In 96 in.?

3. How many feet in 5 yards? In 7 yd.? In 6 yd.? In 12 yd.?

4. How many yards in 12 feet? In 21 ft.? In 18 ft.? In 27 ft.?

5. How many yards in 5 rods?

SOLUTION.—Since in 1 rod there are $5\frac{1}{2}$ yards, in 5 rods there are 5 times $5\frac{1}{2}$ yards. Five times 5 yards are 25 yards, and five times $\frac{1}{2}$ yard = $\frac{5}{2}$ yards, or $2\frac{1}{2}$ yards, which added to 25 yards make $27\frac{1}{2}$ yards.

CONCLUSION.—Therefore, in 5 rods there are $27\frac{1}{2}$ yards.

6. How many yards in 4 rods? In 6 rd.? In 9 rd.?

7. How many rods in 15 yards?

SOLUTION.—Since there are $5\frac{1}{2}$ yards in 1 rod, in 15 yards there are as many rods as $5\frac{1}{2}$ is contained times in 15.

Since 1 yard = 2 halves, $5\frac{1}{2}$ yards = $5\frac{1}{2}$ times 2 halves, which is 11 halves; and 15 yards = 15 times 2 halves, which is 30 halves. 11 halves are contained in 30 halves 2 times, with a remainder of 8 half yards = 4 yards.

CONCLUSION.—Therefore, in 15 yards there are 2 rods and 4 yards.

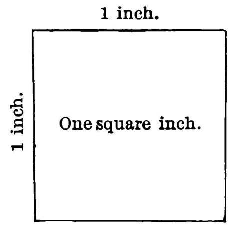
8. How many rods in 25 yards? In 34 yd.? In 44 yd.?

EXAMPLES.

1. Reduce 2 mi. to rods.
2. Reduce 5 yd. 1 ft. 6 in. to inches.
3. Reduce 2340 rd. to miles.
4. Reduce 12 mi. 280 rd. to rods.
5. Reduce 2 yd. 1 ft. 8 in. to inches.
6. Reduce 143808 in. to rods.
7. Reduce 13600 in. to yards.
8. Reduce 5 mi. 138 rd. 3 yd. to feet.
9. A steamboat moves at the rate of 70 feet in a second: how far will it go in 3630 seconds?
10. If a horse can travel 1 mile in 6 minutes, how many yards will he travel in an hour?
11. How many rods from Cincinnati to Columbus, the distance being 120 miles?
12. The circumference of a carriage-wheel is 15 feet: how many times will it revolve in going 7 miles?

69. Land or Square Measure is used in measuring any thing which has both length and breadth.

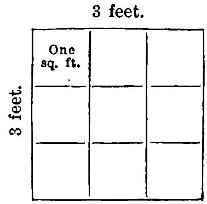
A **Square** is a figure having four equal sides and four right angles, or equal corners. (Teacher explain.)



A **Square Inch** is a square, each side of which is one inch long.

A **Square Foot** is a square, each side of which is a foot long.

A **Square Yard** is a square, each side of which is a yard long.



The figure shows that a square yard, that is, 3 feet square, contains 9 square feet.

By 3 *feet square*, we mean a square, each side of which is three feet; but 3 *square feet* are 3 small squares, each a foot long and a foot wide. 3 feet square contains 9 square feet. (Teacher explain.)

A **Rectangle** is a figure having four sides and four right angles.

The **Area** of a figure is the number of times it contains the unit of measure.

The **Unit of Measure** for surfaces is a square whose side is a *linear* unit; as, a square inch, a square yard.

70. TO FIND THE AREA OF A RECTANGLE.—*Multiply the length by the breadth.*

TO FIND THE LENGTH.—*Divide the area by the breadth.*

TO FIND THE BREADTH.—*Divide the area by the length.*

TABLE.

144 square inches (sq. in.)	make	1 square foot,	marked	sq. ft.
9 square feet	"	1 square yard,	"	sq. yd.
$30\frac{1}{4}$ square yards	"	1 square rd. or perch,	"	sq. rd.
160 square rods	"	1 acre,	"	A.
640 acres	"	1 square mile,	"	sq. mi.

MENTAL EXERCISES.

1. How many square feet in 3 square yards? In 2 sq. yd.? In 5 sq. yd.?

2. How many square yards in 18 square feet? In 45 sq. ft.? In 81 sq. ft.?

3. How many square inches in a piece of paper six inches long and four inches wide?

4. How many square yards in a roof seven yards long and three yards wide?

5. How many square rods in a square piece of ground each side of which measures eight rods?

6. How many square yards in the floor of a room 18 feet long and 15 feet wide?

EXAMPLES.

1. How many square inches in 4 sq. yd. 8 sq. ft. 32 sq. in.?

2. Reduce 186243 square inches to higher denominations.

3. Reduce 3 A. 90 sq. rd. to square rods. 570 sq. rd.

4. Reduce 7800 sq. rd. to acres.

5. Reduce 6482 square inches to higher denominations.

6. How many square inches in an area of 6 square yards 7 square feet?

7. Reduce 14 A. to square rods.

8. Reduce 2 A. 64 sq. rd. to square rods.

9. Reduce 61540 sq. rd. to acres.

10. Reduce 9 A. 20 sq. rd. to square rods.

11. Find the area of a rectangle 14 feet long and 3 feet broad.

12. What is the area of a floor 12 feet long and 9 feet wide?

13. How many square yards in the floor of a hall 42 feet long and 18 feet wide?

14. How many acres in a field 60 rods long and 32 rods wide?

15. What is the area of a gravel walk 30 yards long and 4 feet wide?

NOTE.—Reduce the numbers to the same denomination before multiplying.

16. A board is 10 ft. 6 in. long and 10 in. wide: what is its area?

17. The area of a field is 5 A. 88 sq. rd.; its length is 37 rd.: what is its breadth?

18. The area covered by a house is 72 sq. yd.; its breadth is 24 feet: what is its length?

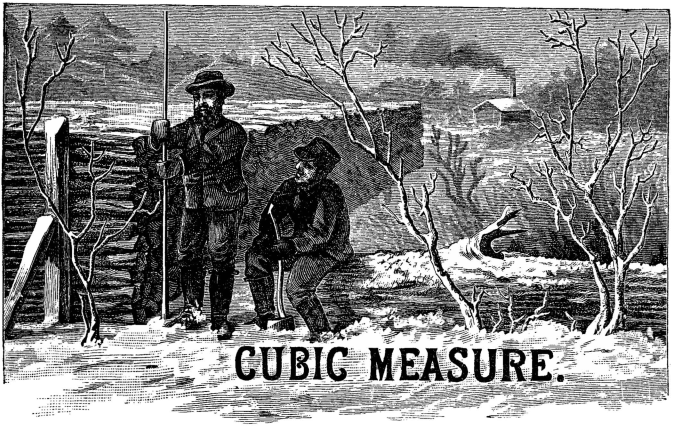
19. The area of a plank is 21 sq. ft.; its breadth is 18 in.: what is its length?

20. How many square yards of plastering does it require to cover one side of a room 15 ft. long and 9 ft. high? How many to cover the two sides? 30 sq. yd.

21. How many square yards of plastering in the walls and ceiling of a room 18 ft. long, 15 ft. wide, and 10 ft. high?

22. How many square yards of carpeting will it take to cover the floor of a room which is 16 ft. long by 12 feet wide?

23. How many square yards of paper will cover the walls of a parlor 21 ft. long, 15 ft. wide, and 12 ft. high? How many rolls of 6 square yards each?



71. Solid or Cubic Measure is used in measuring things having length, breadth, and thickness; such as timber, stone, earth, etc.

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Ray's New Elementary Algebra

DIVISION.

73. Division, in Algebra, is the process of finding how many times one algebraic quantity is contained in another.

Or, having the product of two factors, and one of them given, Division teaches the method of finding the other.

The number by which we divide is called the *divisor;* the number to be divided, the *dividend;* the number of times the divisor is contained in the dividend, the *quotient.*

74. Since the divisor is the known factor and the quotient the one found, their product must always be equal to the dividend.

Division may be indicated by writing the divisor under the dividend in the form of a fraction, or as in arithmetic. Thus, ab divided by a, is written $\frac{ab}{a}$, or $a)ab$.

NOTE.—In solving the following, give the reason for the answer, as in the solution to the first question.

1. How many times is x contained in $4x$? Ans. $\frac{4x}{x}=4$.

$4x$ divided by x, equals 4, because the product of 4 by x is $4x$.

2. How many times is a contained in $6a$? . . Ans.
3. Is a contained in ab? . . . . Ans.
4. Is b contained in $3ab$? . . . . . . Ans.
5. Is 2 contained in $4a$? . . . . . . Ans.
6. Is $2a$ contained in $4ab$? . . . . . Ans.
7. Is a contained in a^3? . . . . . . Ans.
8. Is ab contained in $5a^2b$? . . . . . Ans.
9. Is $4ab^2$ contained in $12a^3b^3c$? . . Ans.
10. Is $2a^2$ contained in $6a^5b$?

SOLUTION, $\frac{6a^5b}{2a^2}=\frac{6}{2}a^{5-2}b=3a^3b$. Ans.

REVIEW.—73. What is Algebraic Division? The divisor? The dividend? The quotient? 74. To what is the product of the quotient and divisor equal? Why? How is division indicated?

In obtaining the quotient, in the foregoing example, we readily see,

1st. That 2 must be multiplied by 3 to produce 6.

2d That a^2 must be multiplied by a^3 to produce a^5; or, we must subtract 2 from 5 to find the exponent of a in the quotient.

3d. That since b is in the dividend, but not in the divisor, it must be in the quotient, so that the product of the divisor and quotient may equal the dividend.

75. It remains to ascertain the rule for the signs.

Since $+a\times+b=+ab$, $-a\times+b=-ab$, $+a\times-b=-ab$, and $-a\times-b=+ab$,

Therefore, $\frac{+ab}{+b}=+a$, $\frac{-ab}{+b}=-a$, $\frac{-ab}{-b}=+a$, and $\frac{+ab}{-b}=-a$.

Or, *like* signs give *plus*, and *unlike* signs give *minus*. Hence,

TO DIVIDE ONE MONOMIAL BY ANOTHER,

Rule.—1. *Divide the coëfficient of the dividend by that of the divisor; observing, that like signs give plus, and unlike signs minus.*

2. *For any letter common to the divisor and dividend, if it has the same exponent in both, suppress it; if not, subtract its exponent in the former from its exponent in the latter, for its exponent in the quotient.*

3. *Annex the letters found in the dividend, but not in the divisor.*

NOTE.—The pupil must recollect that a is the same as a^1.

EXAMPLES.

11. Divide $15a^3bc$ by $3a^2b$. . . . . Ans.
12. Divide $27x^2y^2$ by $-3xy$. . . . . . . Ans.
13. Divide $-18a^3x$ by $-6ax$. . . . . . . Ans.
14. Divide $-12c^4x^3y^5$ by $-4c^4xy^2$. . . . . Ans.
15. Divide $6acx^2y^6v$ by $3ax^2y^4v$. . . . . . . Ans.

REVIEW.—75. When the signs of the dividend and divisor are alike, what will be the sign of the quotient? Why? When unlike? Why? Rule for dividing one monomial by another?

16. Divide $-10c^2x^5y^5v$ by $-2cy^4v$. . . . Ans.
17. Divide $-28ac^2x^8y^4v^2$ by $14ax^5y^4$. . Ans.
18. Divide $30ac^4e^4x^4y^2$ by $-2aex^4$. . . Ans.

NOTE.—The following may be omitted until the book is reviewed:

19. Divide $(x+y)^2$ by $(x+y)$. . . . . Ans.
20. Divide $(a+b)^4$ by $(a+b)$. . . . . Ans.
21. Divide $6(m+n)^3$ by $2(m+n)$. . . Ans.
22. Divide $6a^2b(x+y)^3$ by $2ab(x+y)^2$. Ans.
23. Divide $(x-y)^3(m-n)^2$ by $(x-y)^2(m-n)^2$. Ans.

76. It is evident that one monomial can not be divided by another, in the following cases:

1st. When the coëfficient of the dividend is not exactly divisible by the coëfficient of the divisor.

2d. When the same literal factor has a greater exponent in the divisor than in the dividend.

3d. When the divisor contains one or more literal factors not found in the dividend.

In each of these cases, the division is indicated by writing the divisor under the dividend, in the form of a fraction. This fraction may then be reduced, Art. 129.

77. It has been shown, Art. 68, that any product is multiplied by multiplying either of its factors; hence, conversely, *any dividend will be divided by dividing either of its factors.*

Thus, $\frac{4\times 6}{2}=2\times 6=12$; or, $\frac{4\times 6}{2}=4\times 3=12$.

78. In multiplying a polynomial by a monomial, we multiply each term of the multiplicand by the multiplier. Hence,

REVIEW.—76. In what cases is the exact division of one monomial by another impossible? 78. Rule for dividing a polynomial by a monomial?

TO DIVIDE A POLYNOMIAL BY A MONOMIAL,

Rule.—*Divide each term of the dividend by the divisor, according to the rule for the division of monomials.*

NOTE.—Place the divisor on the left, as in arithmetic.

1. Divide $6x+12y$ by 3. . . . . . . Ans.
2. Divide $15x-20b$ by 5. . . . . . . Ans.
3. Divide $21a+35b$ by -7. . . . Ans.
4. Divide $abc-acf$ by ac. . . . . . Ans.
5. Divide $10ax-15ay$ by $-5a$. . . Ans.
6. Divide $a^2b^2-2ab^3x$ by ab. . . . Ans.
7. Divide $12a^2bc-9acx^2+6ab^2c$ by $-3ac$. Ans.
8. Divide $15a^5b^2c-21a^2b^3c^2$ by $3a^2bc$. Ans.

NOTE.—The following may be omitted until the book is reviewed:

9. Divide $6(a+c)+9(a+x)$ by 3. Ans.
10. Divide $a^2b(c+d)+ab^2(c^2-d)$ by ab. Ans.
11. Divide $ac(m+n)-bc(m+n)$ by $m+n$. Ans.
12. Divide $(m+n)(x+y)^2+(m+n)(x-y)^2$ by $m+n$. Ans.

79. To explain the method of dividing one polynomial by another, we will first find the product of two factors, and then reverse the operation.

Multiplication, or formation of a product.	Division, or decomposition of a product.
$2a^2-ab$	$2a^3-3a^2b+ab^2 \mid a-b$
$a-b$	$2a^3-2a^2b \qquad 2a^2-ab$
$2a^3-a^2b$	1st Rem. $-a^2b+ab^2$
$-2a^2b+ab^2$	$-a^2b+ab^2$
$2a^3-3a^2b+ab^2$	2d Rem. 0

In the foregoing illustration, let the pupil carefully observe, 1st. In the multiplicand, $2a^2—ab$, and the multiplier, $a—b$, a is called the *leading letter*, because its exponents decrease from left to right. It is evident that a will be the leading letter in the product also.

2d. In the division of $2a^3—3a^2b+ab^2$ by $a—b$, the dividend being the product, and the divisor one of the factors, both should be arranged with reference to the same leading letter, in order that the quotient, or remaining factor to be found, may have the same order of arrangement.

3d. If we divide $2a^3$ by a, the result, $2a^2$, will be the term of the quotient by which $a—b$ was first multiplied. If we now multiply $a—b$ by $2a^2$, and subtract the product from the dividend, there will remain $—a^2b+ab^2$, which is the product of $a—b$ by the other term of the quotient. Dividing $—a^2b$ by a, we find this unknown term. Multiplying $a—b$ by it, and subtracting the product, nothing remains.

4th. Had there been a second remainder, the third term of the quotient would have been found in the same manner, and so on for any number of terms.

5th. The divisor is placed on the right of the dividend for convenience in multiplying. Hence,

TO DIVIDE ONE POLYNOMIAL BY ANOTHER,

Rule.—1. *Arrange the dividend and divisor with reference to the leading letter, and place the divisor on the right of the dividend.*

2. *Divide the first term of the dividend by the first term of the divisor, for the first term of the quotient. Multiply the divisor by this term, and subtract the product from the dividend.*

3. *Divide the first term of the remainder by the first term of the divisor, for the second term of the quotient. Multiply the divisor by this term, and subtract the product from the last remainder.*

4. *Proceed in the same manner, and if you obtain* 0 *for a remainder, the division is said to be exact.*

REMARKS.—1. Bring down no more terms of the remainder, at each successive subtraction, than are necessary.

2. It is well to perform the same example in two ways: first, by making the powers of the letter *diminish* from left to right; and, secondly, *increase* from left to right.

3. When the first term of the arranged dividend, or of any remainder, is not exactly divisible by the first term of the arranged divisor, the exact division will be impossible.

1. Divide $6a^2-13ax+6x^2$ by $2a-3x$.

$$\begin{array}{l|l} 6a^2-13ax+6x^2 & 2a-3x \\ \hline 6a^2-9ax & 3a-2x \text{ Quotient.} \\ \hline \quad -4ax+6x^2 & \\ \quad -4ax+6x^2 & \\ \hline \end{array}$$

2. Divide x^2-y^2 by $x-y$.

$$\begin{array}{l|l} x^2-y^2 & x-y \\ \hline x^2-xy & x+y \text{ Quo.} \\ \hline \quad xy-y^2 & \\ \quad xy-y^2 & \\ \hline \end{array}$$

3. Divide a^3+x^3 by $a+x$.

$$\begin{array}{l|l} a^3+x^3 & a+x \\ \hline a^3+a^2x & a^2-ax+x^2 \text{ Quo.} \\ \hline \quad -a^2x+x^3 & \\ \quad -a^2x-ax^2 & \\ \hline \qquad ax^2+x^3 & \\ \qquad ax^2+x^3 & \\ \hline \end{array}$$

4. Divide $5a^2x+5ax^2+a^3+x^3$ by $4ax+a^2+x^2$.

$$\begin{array}{l|l} a^3+5a^2x+5ax^2+x^3 & a^2+4ax+x^2 \\ \hline a^3+4a^2x+ax^2 & a+x \text{ Quotient.} \\ \hline \quad a^2x+4ax^2+x^3 & \\ \quad a^2x+4ax^2+x^3 & \\ \hline \end{array}$$

NOTE.—It is not absolutely necessary to arrange the dividend and divisor with reference to a certain letter; it should be done, however, as a matter of convenience.

In the above example, neither divisor nor dividend being arranged with reference to either a or x, we arrange them with reference to a, and then divide.

REVIEW.—79. What is meant by the leading letter? What is understood by arranging the dividend and divisor with reference to a certain letter? Explain the example given in illustration of division of polynomials.

79. Why is the divisor placed on the right? What is the rule for the division of one polynomial by another? When is the exact division impossible?

5. Divide $a^2+a^3-5a^4+3a^5$ by $a-a^2$.

Both quantities arranged according to the ascending powers of a.

$$\begin{array}{l|l}
a^2+a^3-5a^4+3a^5 & a-a^2 \\ \hline
a^2-a^3 & a+2a^2-3a^3 \\ \cline{1-1}
\quad 2a^3-5a^4 & \text{Quotient.} \\
\quad 2a^3-2a^4 & \\ \cline{1-1}
\qquad -3a^4+3a^5 & \\
\qquad -3a^4+3a^5 & \\ \cline{1-1}
\end{array}$$

Both quantities arranged according to the descending powers of a.

$$\begin{array}{l|l}
3a^5-5a^4+a^3+a^2 & -a^2+a \\ \hline
3a^5-3a^4 & -3a^3+2a^2+a \\ \cline{1-1}
\quad -2a^4+a^3 & \text{Quotient.} \\
\quad -2a^4+2a^3 & \\ \cline{1-1}
\qquad -a^3+a^2 & \\
\qquad -a^3+a^2 & \\ \cline{1-1}
\end{array}$$

It will be seen that the two quotients are the same, but differently arranged. If preferred, the divisor may be placed on the *left*, instead of on the *right*, of the dividend.

6. Divide $4a^2-8ax+4x^2$ by $2a-2x$. Ans.
7. Divide $2x^2+7xy+6y^2$ by $x+2y$. Ans.
8. Divide $2mx+3nx+10mn+15n^2$ by $x+5n$. Ans.
9. Divide $x^2+2xy+y^2$ by $x+y$. . . . . Ans.
10. Divide $8a^4-8x^4$ by $2a^2-2x^2$. . . Ans.
11. Divide $ac+bc-ad-bd$ by $a+b$. . . Ans.
12. Divide $x^3+y^3+5xy^2+5x^2y$ by $x^2+4xy+y^2$. Ans.
13. Divide $a^3-9a^2+27a-27$ by $a-3$. Ans.
14. Divide $4a^4-5a^2x^2+x^4$ by $2a^2-3ax+x^2$. Ans.
15. Divide x^4-y^4 by $x-y$. . . Ans.
16. Divide a^3-b^3 by a^2+ab+b^2. . . Ans.
17. Divide $x^3-y^3+3xy^2-3x^2y$ by $x-y$. Ans.
18. Divide $4x^4-64$ by $2x-4$. Ans.
19. Divide $a^5-5a^4x+10a^3x^2-10a^2x^3+5ax^4-x^5$ by $a^2-2ax+x^2$. Ans.
20. Divide $4a^6-25a^2x^4+20ax^5-4x^6$ by $2a^3-5ax^2+2x^3$. Ans.
21. Divide y^3+1 by $y+1$. . . . . Ans.

22. Divide $6a^4+4a^3x-9a^2x^2-3ax^3+2x^4$ by $2a^2+2ax-x^2$. Ans.

23. Divide $3a^4-8a^2b^2+3a^2c^2+5b^4-3b^2c^2$ by a^2-b^2. Ans.

24. Divide $x^6-3x^4y^2+3x^2y^4-y^6$ by $x^3-3x^2y+3xy^2-y^3$. Ans.

MISCELLANEOUS EXERCISES.

1. $3a+5x-9c+7d+5a-3x-3d-(4a+2x-8c+4d)=$what? Ans.

2. $a+b-(2a-3b)-(5a+7b)-(-13a+2b)=$what? Ans.

3. $(a+b)(a+b)+(a-b)(a-b)=$what? Ans.

4. $(a^2+a^4+a^6)(a^2-1)-(a^4+a)(a^4-a)=$what? Ans.

5. $(a^3+a^2b-ab^2-b^3)\div(a-b)-(a-b)(a-b)=$what? Ans.

II. ALGEBRAIC THEOREMS,

DERIVED FROM MULTIPLICATION AND DIVISION.

80. If we square $a+b$, that is, multiply $a+b$ by itself, the product will be $a^2+2ab+b^2$.

Thus:
$$\begin{array}{r l} & a+b \\ & a+b \\ \hline & a^2+ab \\ & \quad\; +ab+b^2 \\ \hline & a^2+2ab+b^2 \end{array}$$

But $a+b$ is the sum of the quantities, a and b. Hence,

Theorem I.—*The square of the sum of two quantities is equal to the square of the first, plus twice the product of the first by the second, plus the square of the second.*

NOTE.—Let the pupil apply the theorem by writing the following examples, enunciated thus: What is the square of 2+3?

1. $(2+3)^2=4+12+9=25$.
2. $(2a+b)^2=4a^2+4ab+b^2$.
3. $(2x+3y)^2=4x^2+12xy+9y^2$.
4. $(ab+cd)^2=a^2b^2+2abcd+c^2d^2$.
5. $(x^2+xy)^2=x^4+2x^3y+x^2y^2$.
6. $(2a^2+3ax)^2=4a^4+12a^3x+9a^2x^2$.

81. If we square $a-b$, that is, multiply $a-b$ by itself, the product will be $a^2-2ab+b^2$.

$$\begin{array}{lrl} \text{Thus:} & a-b & \\ & a-b & \\ \hline & a^2-ab & \\ & -ab & +b^2 \\ \hline & a^2-2ab & +b^2 \end{array}$$

But $a-b$ is the difference of the quantities, a and b. Hence,

Theorem II.—*The square of the difference of two quantities is equal to the square of the first, minus twice the product of the first by the second, plus the square of the second.*

1. $(5-4)^2=25-40+16=1$.
2. $(2a-b)^2=4a^2-4ab+b^2$.
3. $(3x-2y)^2=9x^2-12xy+4y^2$.
4. $(x^2-y^2)^2=x^4-2x^2y^2+y^4$.
5. $(ax-x^2)^2=a^2x^2-2ax^3+x^4$.
6. $(5a^2-b^2)^2=25a^4-10a^2b^2+b^4$.

82. If we multiply $a+b$ by $a-b$, the product will be a^2-b^2.

$$\begin{array}{lrl} \text{Thus:} & a+b & \\ & a-b & \\ \hline & a^2+ab & \\ & -ab & -b^2 \\ \hline & a^2 & -b^2 \end{array}$$

But $a+b$ represents the sum of two quantities, and $a-b$, their difference. Hence,

Theorem III.—*The product of the sum and difference of two quantities is equal to the difference of their squares.*

1. $(5+3)(5-3)=25-9=16=8\times2$.
2. $(2a+b)(2a-b)=4a^2-b^2$.
3. $(2x+3y)(2x-3y)=4x^2-9y^2$.
4. $(5a+4b)(5a-4b)=25a^2-16b^2$.
5. $(a^2+b^2)(a^2-b^2)=a^4-b^4$.
6. $(2am+3bn)(2am-3bn)=4a^2m^2-9b^2n^2$.

83. If we divide a^3 by a^5, observing the rule for the exponents, we have $\frac{a^3}{a^5}=a^{3-5}=a^{-2}$. But, Art. 127, $\frac{a^3}{a^5}=\frac{1}{a^2}$.

So, $\frac{a^m}{a^n}=a^{m-n}$; and $\frac{a^m}{a^n}=\frac{1}{a^{n-m}}$. Hence, $a^{m-n}=\frac{1}{a^{n-m}}$.

Also, $\frac{a}{b^m}=a\times\frac{1}{b^m}=ab^{-m}$; $\frac{a^m}{b^n}=a^mb^{-n}$; $\frac{a}{b}=ab^{-1}$; $\frac{1}{ab^2}=a^{-1}b^{-2}$; $\frac{ab^3}{c}=\frac{a}{b^{-3}c}$, etc. Hence,

Theorem IV.—1. *The reciprocal of a quantity is equal to the same quantity with the sign of its exponent changed.*

2. *Any quantity may be transferred from one term of a fraction to the other, if the sign of the exponent be changed.*

Thus: . . . $\frac{a}{b}=ab^{-1}=\frac{b^{-1}}{a^{-1}}=\frac{1}{a^{-1}b}$;

$$\frac{a^2b^2}{c^2d^3}=a^2b^2c^{-2}d^{-3}=\frac{1}{a^{-2}b^{-2}c^2d^3}=\frac{c^{-2}d^{-3}}{a^{-2}b^{-2}}.$$

84. By the rule for the exponents, Art. 74, $\frac{a^2}{a^2}=a^{2-2}=a^0$; but since any quantity is contained in itself once, $\frac{a^2}{a^2}=1$.

Similarly, $\frac{a^m}{a^m}=a^{m-m}=a^0$; but $\frac{a^m}{a^m}=1$; therefore, $a^0=1$, since each is equal to $\frac{a^m}{a^m}$. Hence,

Theorem V.—*Any quantity whose exponent is 0 is equal to unity.*

85. If we divide a^2-b^2 by $a-b$, the quotient will be $a+b$. If we divide a^3-b^3 by $a-b$, the quotient will be a^2+ab+b^2.

In the same manner, we should find, by trial, that the quotients obtained by dividing the difference of the same powers of two quantities by the difference of those quantities, follow a simple law.

Thus: $(a^2-b^2)\div(a-b)=a+b.$
$(a^3-b^3)\div(a-b)=a^2+ab+b^2.$
$(a^4-b^4)\div(a-b)=a^3+a^2b+ab^2+b^3.$
$(a^5-b^5)\div(a-b)=a^4+a^3b+a^2b^2+ab^3+b^4.$

So, $(a^5-1)\div(a-1)=a^4+a^3+a^2+a+1.$
And $(1-b^5)\div(1-b)=1+b+b^2+b^3+b^4.$

The exponent of the first letter decreases by unity, while that of the second increases by unity. Hence, we have

Theorem VI.—*The difference of the same powers of two quantities is always divisible by the difference of the quantities.*

86. The two following theorems may also be readily shown to be true by trial:

Theorem VII.—*The difference of the even powers of two quantities of the same degree, is always divisible by the sum of the quantities.*

Thus: $(a^2-b^2)\div(a+b)=a-b.$
$(a^4-b^4)\div(a+b)=a^3-a^2b+ab^2-b^3.$
$(a^6-b^6)\div(a+b)=a^5-a^4b+a^3b^2-a^2b^3+ab^4-b^5.$

So, $(a^6-1)\div(a+1)=a^5-a^4+a^3-a^2+a-1.$
And $(1-b^6)\div(1+b)=1-b+b^2-b^3+b^4-b^5.$

Review.—80. To what is the square of the sum of two quantities equal? 81. Of the difference of two quantities? 82. The product of the sum and difference of two quantities?

83. How may the reciprocal of any quantity be expressed? How may any quantity be transferred from one term of a fraction to the other? In what other form may a^m be written? a^{-m}?

84. What is the value of any quantity whose exponent is zero?

ECLECTIC EDUCATION SERIES

Ray's New Higher Arithmetic

XII. RATIO.

DEFINITIONS.

228. 1. **Ratio** is a Latin word, signifying *relation* or *connection;* in Arithmetic, it is *the measure of the relation of one number to another of the same kind, expressed by their quotient.*

2. A Ratio is found by dividing the first number by the second; as, the ratio of 8 to 4 is 2. The ratio is abstract.

3. The **Sign of Ratio** is the colon (:), which is the sign of division, with the horizontal line omitted; thus, 6 : 4 signifies the ratio of 6 to $4 = \frac{6}{4}$.

4. Each number is called a term of the ratio, and both together a **couplet** or **ratio.** The first term of a ratio is the **antecedent,** which means *going before;* the second term is the **consequent,** which means *following.*

5. A **Simple Ratio** is a single ratio consisting of two terms; as, $3 : 4 = \frac{3}{4}$.

6. A **Compound Ratio** is the product of two or more simple ratios; as, $\left\{\begin{matrix} 3 : 7 \\ 5 : 8 \end{matrix}\right\} = \frac{3 \times 5}{7 \times 8}$.

7. The **Reciprocal of a Ratio** is 1 divided by the ratio, or the ratio inverted; thus, the reciprocal of 2 : 3, or $\frac{2}{3}$, is $1 \div \frac{2}{3} = \frac{3}{2}$.

8. **Inverse Ratio** is the quotient of the consequent divided by the antecedent; thus, $\frac{5}{4}$ is the inverse ratio of 4 to 5.

9. The **Value of the Ratio** depends upon the relative size of the terms.

229. From the preceding definitions the following principles are derived:

PRINCIPLES.—1. $Ratio = \frac{Antecedent}{Consequent}$.

2. $Antecedent = Consequent \times Ratio$.

3. $Consequent = \frac{Antecedent}{Ratio}$.

Hence, by Art. **87**:

1. *The Ratio is multiplied by multiplying the Antecedent or dividing the Consequent.*

2. *The Ratio is divided by dividing the Antecedent or multiplying the Consequent.*

3. *The Ratio is not changed by multiplying or dividing both terms by the same number.*

General Law.—*Any change in the Antecedent produces a like change in the Ratio, but any change in the Consequent produces an opposite change in the Ratio.*

PROBLEM.—What is the ratio of 15 to 36?

OPERATION.

$$15 : 36 = \frac{15}{36} = \frac{5}{12}.$$

Rule.—*Divide the Antecedent by the Consequent.*

EXAMPLES FOR PRACTICE.

1. What is the ratio of 2 ft. 6 in. to 3 yd. 1 ft. 10 in.?

2. What is the ratio of 4 mi. 260 rd. to 1 mi. 96 rd.?

3. What is the ratio of 13 A. 145 sq. rd. : 6 A. 90 sq. rd.?

4. What is the ratio of 3 ℔. 10 oz. 6 pwt. $10\frac{1}{2}$ gr. : 2 ℔. $14\frac{3}{4}$ pwt.?

5. What is the ratio of 10 gal. 1.54 pt. : 7 gal. 2 qt. .98 pt.?

6. What is the ratio of 56 bu. 2 pk. 1 qt. : 35 bu. 3 pk. 6.055 qt.?

7. If the antecedent is 7 and the ratio $1\frac{1}{2}$, what is the consequent?

8. If the consequent is $\frac{3}{7}$ and the ratio $\frac{2}{9}$, what is the antecedent?

9. What is the ratio of a yard to a meter, and of a meter to a yard?

10. What is the ratio of a pound avoirdupois to a pound troy?

11. Find the difference between the compound ratios $\left\{\begin{matrix} 3 : 4 \\ 5 : 9 \end{matrix}\right\}$ and $\left\{\begin{matrix} 1 : 6 \\ 2 : 7 \end{matrix}\right\}$.

12. Find the difference between the ratio $4\frac{2}{3} : 7\frac{1}{2}$ and the inverse ratio.

13. If the consequent is $6\frac{1}{2}$, and the ratio is $2\frac{1}{4}$, what is the antecedent, and what is the inverse ratio of the two numbers?

XIII. PROPORTION.

DEFINITIONS.

230. 1. **Proportion** is an equality of ratios.

Thus, 4 : 6 : : 8 : 12 is a proportion, and is read 4 *is to* 6 *as* 8 *is to* 12.

2. The **Sign of Proportion** is the double colon (: :).

NOTE.—It is the same in effect as the sign of equality, which is sometimes used in its place.

3. The two ratios compared are called **couplets.** The first couplet is composed of the first and second terms, and the second couplet of the third and fourth terms.

4. Since each ratio has an antecedent and consequent, every proportion has *two* antecedents and *two* consequents, the 1st and 3d terms being the antecedents, and the 2d and 4th the consequents.

5. The first and last terms of a proportion are called the **extremes;** the middle terms, the **means.** All the terms are called **proportionals,** and the last term is said to be a *fourth proportional* to the other three in their order.

6. When three numbers are proportional, the second number is a **mean proportional** between the other two.

Thus, 4 : 6 : : 6 : 9; six is a mean proportional between 4 and 9.

7. Proportion is either **Simple** or **Compound:** Simple when both ratios are simple; Compound when one or both ratios are compound.

PRINCIPLES.—1. *In every proportion the product of the means is equal to the product of the extremes.*

2. *The product of the extremes divided by either mean, will give the other mean.*

3. *The product of the means divided by either extreme, will give the other extreme.*

SIMPLE PROPORTION.

231. 1. **Simple Proportion** is an expression of equality between *two simple ratios.*

2. It is employed when three terms are given and we wish to find the fourth. Two of the three terms are alike, and the other is of the same kind as the fourth which is to be found.

3. All proportions must be true according to Principle 1, which is the test. Principles 2 and 3 indicate methods of finding the wanting term.

4. The **Statement** is the proper arrangement of the terms of the proportion.

PROBLEM.—If 6 horses cost $300, what will 15 horses cost?

STATEMENT.

6 horses : 15 horses :: $300 : ($).

OPERATION.

$$\frac{\$300\times15}{6}=\$750. \quad \text{Or, } (\$300\times15)\div6=\$750, \textit{Ans.}$$

SOLUTION.—Since 6 horses and 15 horses may be compared, they form the first couplet; also, $300 and $ — may be compared, as they are of the same unit of value.

NOTES.—1. To find the missing extreme, we use Prin. 3.

2. To prove the proportion, we use Prin. 1. Thus, $\$750\times6=\$300\times15$.

PROBLEM.—If 15 men do a piece of work in $9\frac{3}{5}$ da., how long will 36 men be in doing the same?

SOLUTION.—Since 36 men will require *less* time than 15 men to do the same work, the answer should be *less* than $9\frac{3}{5}$ da.; make a decreasing ratio, $\frac{15}{36}$, and multiply the remaining quantity by it.

STATEMENT.

men men da. da.

$36 : 15 :: 9\frac{3}{5} : (\ )$

OPERATION.

$9\frac{3}{5}=\frac{48}{5}$ da.

$\frac{48}{5}\times\frac{15}{36}=4$ da., *Ans.*

Rule.—1. *For the third term, write that number which is of the same denomination as the number required.*

2. *For the second term, write the* GREATER *of the two remaining numbers, when the fourth term is to be greater than the third; and the* LESS, *when the fourth term is to be less than the third.*

3. *Divide the product of the second and third terms by the first; the quotient will be the fourth term, or number required.*

Examples for Practice.

Note.—Problems marked with an asterisk are to be solved mentally.

1.* If I walk $10\frac{1}{2}$ mi. in 3 hr., how far will I go in 10 hr., at the same rate?

2. If the fore-wheel of a carriage is 8 ft. 2 in. in circumference, and turns round 670 times, how often will the hind-wheel, which is 11 ft. 8 in. in circumference, turn round in going the same distance?

3. If a horse trot 3 mi. in 8 min. 15 sec., how far can he trot in an hour, at the same rate?

4. What is a servant's wages for 3 wk. 5 da., at $1.75 per week?

5. What should be paid for a barrel of powder, containing 132 lb., if 15 lb. are sold for $\$5.43\frac{3}{4}$?

6. A body of soldiers are 42 in rank when they are 24 in file: if they were 36 in rank, how many in file would there be?

7. If a pulse beats 28 times in 16 sec., how many times does it beat in a minute?

8. If a cane 3 ft. 4 in. long, held upright, casts a shadow 2 ft. 1 in. long, how high is a tree whose shadow at the same time is 25 ft. 9 in.?

9. If a farm of 160 A. rents for $450, how much should be charged for one of 840 A?

10. A grocer has a false gallon, containing 3 qt. $1\frac{1}{2}$ pt.: what is the worth of the liquor that he sells for $240, and what is his gain by the cheat?

11. If he uses $14\frac{3}{4}$ oz. for a pound, how much does he cheat by selling sugar for $27.52?

12. An equatorial degree is 365000 ft.: how many ft. in $80° 24' 37''$ of the same?

13. If a pendulum beats 5000 times a day, how often does it beat in 2 hr. 20 min. 5 sec.?

14.* If it takes 108 days, of $8\frac{1}{2}$ hr., to do a piece of work, how many days of $6\frac{3}{4}$ hr. would it take?

15. A man borrows $1750, and keeps it 1 yr. 8 mon.: how long should he lend $1200 to compensate for the favor?

16. A garrison has food to last 9 mon., giving each man 1 lb. 2 oz. a day: what should be a man's daily allowance, to make the same food last 1 yr. 8 mon.?

17. A garrison of 560 men have provisions to last during a siege, at the rate of 1 lb. 4 oz. a day per man; if the daily allowance is reduced to 14 oz. per man, how large a reinforcement could be received?

18. A shadow of a cloud moves 400 ft. in $18\frac{3}{4}$ sec.: what was the wind's velocity per hour?

19. If 1 ℔. troy of English standard silver is worth £3 6s., what is 1 lb. av. worth?

20. If I go a journey in $12\frac{3}{4}$ days, at 40 mi. a day, how long would it take me at $29\frac{3}{4}$ mi. a day?

21.* If $\frac{5}{9}$ of a ship is worth $6000, what is the whole of it worth?

22. If A, worth $5840, is taxed $78.14, what is B worth, who is taxed $256.01?

23.* What are 4 lb. 6 oz. of butter worth, at 28 ct. a lb.?

24. If I gain $160.29 in 2 yr. 3 mon., what would I gain in 5 yr. 6 mon., at that rate?

25. If I gain $92.54 on $1156.75 worth of sugar, how much must I sell to gain $67.32?

26. If coffee costing $255 is now worth $318.75, what did $1285.20 worth cost?

27. A has cloth at $3.25 a yd., and B has flour at $5.50 a barrel. If, in trading, A puts his cloth at $3.62\frac{1}{2}$, what should B charge for his flour?

28.* If a boat is rowed at the rate of 6 miles an hour, and is driven 44 feet in 9 strokes of the oar, how many strokes are made in a minute?

29. If I gain \$7.75 by trading with \$100, how much ought I to gain on \$847.56?

30. What is a pile of wood, 15 ft. long, $10\frac{1}{2}$ ft. high, and 12 ft. wide, worth, at \$4.25 a cord?

REMARK.—In Fahrenheit's thermometer, the freezing point of water is marked 32°, and the boiling point 212°: in the Centigrade, the freezing point is 0°, and the boiling point 100°: in Reaumer's, the freezing point is 0°, and the boiling point 80°.

31. From the above data, find the value of a degree of each thermometer in the degrees of the other two.

32. Convert 108° F. to degrees of the other two thermometers.

33. Convert 25° R. to degrees of the other two thermometers.

34. Convert 46° C. to degrees of the other two thermometers.

REMARKS.—1. In the working of machinery, it is ascertained that *the available power is to the weight overcome, inversely as the distances they pass over in the same time.*

2. *Inverse variation* exists between two numbers when one increases as the other decreases.

3. The *available* power is taken $\frac{2}{3}$ of the whole power, $\frac{1}{3}$ being allowed for friction and other impediments.

35. If the whole power applied is 180 lb. and moves 4 ft., how far will it lift a weight of 960 lb.?

36. If 512 lb. be lifted 1 ft. 3 in. by a power moving 6 ft. 8 in., what is the power?

37. A lifts a weight of 1440 lb. by a wheel and axle; for every 3 ft. of rope that passes through his hands the weight rises $4\frac{1}{2}$ in.: what power does he exert?

38. A man weighing 198 lb. lets himself down 54 ft. with a uniform motion, by a wheel and axle: if the weight at the hook rises 12 ft., how much is it?

39. Two bodies free to move, attract each other with forces that vary inversely as their weights. If the weights are 9 lb. and 4 lb., and the smaller is attracted 10 ft., how far will the larger be attracted?

40. Suppose the earth and moon to approach each other in obedience to this law, their weights being 49147 and 123 respectively, how many miles would the moon move while the earth moved 250 miles?

Can the three following questions be solved by proportion?

41. If 3 men mow 5 A. of grass in a day, how many men will mow $13\frac{1}{3}$ A. in a day?

42.* If 6 men build a wall in 7 da., how long would 10 men be in doing the same?

43.* If I gain 15 cents each, by selling books at $4.80 a doz., what is my gain on each at $5.40 a doz.?

44. A clock which loses 5 minutes a day, was set right at 6 in the morning of January 1st: what will be the right time when that clock points to 11 on the 15th?

45. If water begin and continue running at the rate of 80 gal. an hour, into a cellar 12 ft. long, 8 ft. wide, and 6 ft. deep, while it soaks away at the rate of 35 gal. an hour, in what time will the cellar be full?

46. Take the proportion of 4 : 9 :: 252 : a fourth term. If the third and fourth terms each be increased by 7, while the first remains unchanged, what multiplier is needed by the second to make a proportion?

47. Prove that there is no number which can be added to each term of 6 : 3 :: 18 : 9 so that the resulting numbers shall stand in proportion.

48. A certain number has been divided by one more than itself, giving a quotient $\frac{1}{5}$: what is the number?

49. If 48 lb. of sea-water contain $1\frac{1}{2}$ lb. of salt, how much fresh water must be added to these 48 lb. so that 40 lb. of the mixture shall contain $\frac{1}{2}$ lb. of salt?

COMPOUND PROPORTION.

232. Compound Proportion is an expression of equality between two ratios when either or when each ratio is Compound.

PROBLEM.—If 3 men mow 8 A. of grass in 4 da., how long would 10 men be in mowing 36 A.?

STATEMENT.

$$\left.\begin{array}{l}10 \text{ men} : 3 \text{ men}\\ 8 \text{ A.} \quad : 36 \text{ A.}\end{array}\right\} :: 4 \text{ da.} : (\ ) \text{ da.}$$

OPERATION.

$$\frac{3 \times \overset{\overset{9}{\cancel{18}}}{\cancel{36}} \times \cancel{4}}{\underset{5}{\cancel{10}} \times \underset{2}{\cancel{8}}} = \frac{27}{5} = 5\tfrac{2}{5} \text{ da. } \textit{Ans.}$$

SOLUTION.—Since the denomination of the required term is days, make the third term 4 da. In forming the first and second terms, consider each denomination separately; 10 men can do the same amount of work in less time than 3 men; hence, the first ratio is, 10 men : 3 men, the *less* number being the second term. Since it takes 4 da. to mow 8 A., it will take a greater number of days to mow 36 A., and the second ratio is, 8 A. : 36 A., the *greater* number being the second term. Then dividing the continued product of the means by that of the extremes (Art. **230**, Prin. 3), after cancellation, we have $5\frac{2}{5}$ da., the required term.

Rule.—1. *For the third term, write that number which is of the same denomination as the number required.*

2. *Arrange each pair of numbers having the same denomination in the compound ratio, as if, with the third term, they formed a simple proportion.*

3. *Divide the product of the numbers in the second and third terms by the product of the numbers in the first term: the quotient will be the required term.*

233. Problems in Compound Proportion are readily solved by separating all the quantities involved into *two causes* and *two effects.*

PROBLEM.—If 6 men, in 10 days of 9 hr. each, build 25 rd. of fence, how many hours a day must 8 men work to build 48 rd. in 12 days?

SOLUTION.—6 men 10 da. and 9 hr. constitute the first cause, whose effect is 25 rd.; 8 men 12 da. and ( ) hr. constitute the second cause, whose effect is 48 rd. Hence,

STATEMENT.

$$\left.\begin{matrix}6\text{ men.}\\10\text{ da.}\\9\text{ hr.}\end{matrix}\right\}:\left.\begin{matrix}8\text{ men.}\\12\text{ da.}\\(\ )\text{ hr.}\end{matrix}\right\}::25\text{ rd.}:48\text{ rd.}$$

OPERATION.

$$\frac{6\times \overset{5}{\cancel{10}}\times 9\times \overset{4}{\cancel{48}}}{\underset{2}{\cancel{8}}\times \cancel{12}\times \underset{5}{\cancel{25}}}=\frac{54}{5}=10\tfrac{4}{5}\text{ hr. } Ans.$$

Rule of Cause and Effect.—1. *Separate all the quantities contained in the question into two causes and their effects.*

2. *Write, for the first term of a proportion, all the quantities that constitute the first cause; for the second term, all that constitute the second cause; for the third, all that constitute the effect of the first cause; and for the fourth, all that constitute the effect of the second cause.*

3. *The required quantity may be indicated by a bracket, and found by* Art. **230,** Principles.

NOTE.—The two causes must be exactly alike in the *number* and *kind* of their terms; and so must the two effects.

EXAMPLES FOR PRACTICE.

1. If 18 pipes, each delivering 6 gal. per minute, fill a cistern in 2 hr. 16 min., how many pipes, each delivering 20 gal. per minute, will fill a cistern $7\frac{1}{2}$ times as large as the first, in 3 hr. 24 min.?